



יסודות בקרה במרחב המצב (00860721)

דוגמאות שאלות מבחינות סופיות

3 באוגוסט 2026

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Part I

Linear algebra

Question 1

What is the rank of $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 1 \end{bmatrix}$?

rank $A =$ _____

Question 2

Which matrix below may be $\exp\left(\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} t\right)$ for some $t \in \mathbb{R}$

$$A_1 = \begin{bmatrix} 3 & 3 \\ 1 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 2 & 0 \\ 3 & 1 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}, \quad A_4 = \begin{bmatrix} 8 & 0 \\ 0 & 1 \end{bmatrix}?$$

A _____, because _____

Question 3

Is $(A - I)^4 = 0$ for $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 5 & 1 \end{bmatrix}$?

yes/no, because _____

Question 4

Calculate

$$\begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}^{18} = \underline{\hspace{2cm}}$$

Question 5

Calculate

$$\exp\left(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} t\right) = \underline{\hspace{2cm}}$$

Question 6

Is $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} > 0$?

yes/no, because _____

Question 7

$$\text{Is } \begin{bmatrix} 14 & 1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 \\ 3 & 0 & 0 & 1 \end{bmatrix} > 0?$$

yes/no, because

Question 8

$$\text{Is } \begin{bmatrix} -2 & 0 & 0 & 2 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 1 \\ 2 & 1 & 1 & -5 \end{bmatrix} < 0?$$

yes/no, because

Question 9

$$\text{Suppose } X > 0. \text{ Is } \begin{bmatrix} -X & I \\ I & -X^{-1} \end{bmatrix} < 0?$$

yes/no, because

Question 10

$$\text{Does } \begin{bmatrix} -\gamma I & D \\ D' & -\gamma I \end{bmatrix} < 0 \text{ imply that } \|D\| < \gamma?$$

yes/no, because

Question 11

Is the equation

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} X + X \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix} + Q = 0$$

solvable in $X \in \mathbb{R}^{2 \times 3}$ for all $Q \in \mathbb{R}^{2 \times 3}$?

yes/no, because

Question 12

Is the equation

$$Y = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} Y \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix} + Q$$

solvable in $Y \in \mathbb{R}^{2 \times 3}$ for all $Q \in \mathbb{R}^{2 \times 3}$?

yes/no, because

Question 13

Is the solution X to

$$\begin{bmatrix} -1 & 10 \\ 0 & -2 \end{bmatrix} X + X \begin{bmatrix} -1 & 0 \\ 10 & -2 \end{bmatrix} + I = 0$$

positive definite?

yes/no, because

Part II

Properties of state-space realizations

Question 14

What is the pole excess q of the transfer function of a system, whose state-space realization is

$$(A, B, C, D) = \left(\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ -2 \end{bmatrix}, [1 \ 0 \ 0], 1 \right) ?$$

$q =$ _____, because

Question 15

What is the pole excess q of the transfer function of a system, whose state-space realization is

$$(A, B, C, D) = \left(\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ -2 \end{bmatrix}, [1 \ 0 \ 0], 0 \right) ?$$

$q =$ _____, because

Question 16*

Consider a minimal order- n realization (A, B, C, D) of a SISO system G and assume that

$$\text{rank} \begin{bmatrix} A - \lambda I & B \\ C & D \end{bmatrix} = n + 1, \quad \forall \lambda \in \mathbb{C}$$

What is the pole excess q of $G(s)$?

$q =$ _____, because

Question 17*

Consider an order- n realization $(A, B, C, 0)$ of an LTI system G . Prove that if $CA^i B = 0$ for all $i \in \mathbb{Z}_{0..n-1}$, then $G = 0$.

Question 18

Can the realizations $\left(\begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, [1 \ -1], 1\right)$ and $\left(\begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, [1 \ 0], 0\right)$ be similar?

yes / no, because

Question 19

Can the realizations $\left(\begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, [1 \ -1], 0\right)$ and $\left(\begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, [1 \ 0], 0\right)$ be similar?

yes / no, because

Question 20

Can the realizations $\left(\begin{bmatrix} 2 & 0 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, [1 \ 0], 0\right)$ and $\left(\begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, [1 \ 0], 0\right)$ be similar?

yes / no, because

Question 21

Given $G : \begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{cases}$. Derive a state-space realization of $2G + I$ whose dimension matches that of G .

Question 22

Given $G : \begin{cases} x[t+1] = Ax[t] + Bu[t] \\ y[t] = Cx[t] + 2u[t] \end{cases}$. Derive a state-space realization of $(G - I)^{-1}$ whose dimension matches that of G .

Question 23

Consider a second-order system of the form $\dot{x}(t) = Ax(t) + Bu(t)$ and assume that there is a control law attaining $x(5) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ from all $x(0)$. Does it imply that the system controllable?

yes / no, because

Question 24

Consider a second-order system of the form $\dot{x}(t) = Ax(t) + Bu(t)$ and assume that there is a control law attaining $x(5) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ from $x(0) = 0$. Does it imply that the system controllable?

yes / no, because

Question 25

Is the system $\dot{x}(t) = -x(t) + \begin{bmatrix} 1 \\ 2 \end{bmatrix}u(t)$ controllable?

yes / no, because

Question 26

Is the system $\dot{x}(t) = -x(t) + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u(t)$ stabilizable ?

yes / no, because

Question 27

Is the system $\dot{x}(t) = \begin{bmatrix} 2 & 0 & -1 \\ 0 & -5 & 4 \\ 0 & 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u(t)$ stabilizable ?

yes / no, because

Question 28

Is the system $\begin{cases} \dot{x}(t) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} x(t) \\ y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t) \end{cases}$ observable ?

yes / no, because

Question 29

Is the system $\begin{cases} \dot{x}(t) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} x(t) \\ y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t) \end{cases}$ detectable ?

yes / no, because

Question 30

Is the system $x[t + 1] = \begin{bmatrix} 1 \\ 2 \end{bmatrix} u[t]$ reachable ?

yes / no, because

Question 31

Is the system $x[t + 1] = \begin{bmatrix} 1 \\ 2 \end{bmatrix} u[t]$ controllable ?

yes / no, because

Question 32

Is the system $x[t + 1] = \begin{bmatrix} 1 \\ 2 \end{bmatrix} u[t]$ stabilizable ?

yes / no, because

Question 33

What mode of $\left(\begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}\right)$ is uncontrollable?

$\lambda =$ _____, because _____

Question 34

Is the solution X to

$$\begin{bmatrix} -1 & 10 \\ 0 & -2 \end{bmatrix} X + X \begin{bmatrix} -1 & 0 \\ 10 & -2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} = 0$$

positive definite?

yes/no, because _____

Question 35

Is

$$\int_0^8 \exp\left(\begin{bmatrix} 0 & 1 \\ -2 & 2 \end{bmatrix} s\right) \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \exp\left(\begin{bmatrix} 0 & -2 \\ 1 & 2 \end{bmatrix} s\right) ds > 0?$$

yes/no, because _____

Question 36

Is the solution X to

$$X = \begin{bmatrix} -1 & 10 \\ 0 & -2 \end{bmatrix} X \begin{bmatrix} -1 & 0 \\ 10 & -2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix}$$

positive definite?

yes/no, because _____

Question 37

Is there $t \in \mathbb{N}$ such that

$$\sum_{s=0}^t \begin{bmatrix} 0 & 1 \\ -2 & 2 \end{bmatrix}^s \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -2 \\ 1 & 2 \end{bmatrix}^s > 0?$$

yes/no, because _____

Question 38

Can $\bar{A} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$ and $\bar{B} = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}$ be the matrices of the stroboscopic discretization of $\dot{x}(t) = x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$ for some sampling period $h > 0$?

yes/no, because _____

Question 39

Can $\bar{A} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ and $\bar{B} = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}$ be the matrices of the stroboscopic discretization of $\dot{x}(t) = x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$ for some sampling period $h > 0$?

yes / no, because

Question 40

Can the stroboscopic discretization of $\dot{x}(t) = \begin{bmatrix} 0 & \pi \\ -\pi & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$ have $\text{spec } \bar{A} = \{1, 1\}$ for some sampling period $h > 0$?

yes / no, because

Question 41

Is the realization $(\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \end{bmatrix}, 0)$ minimal?

yes / no, because

Question 42

Is the realization $(\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \end{bmatrix}, 1)$ minimal?

yes / no, because

Question 43

What is the H_∞ -norm of a system G , whose transfer function $G(s) = \left[\begin{array}{c|c} -1 & 1 \\ \hline 1 & 0 \end{array} \right]$?

$\|G\|_\infty =$ _____, because

Question 44

What is the H_∞ -norm of a system G , whose transfer function $G(s) = \left[\begin{array}{cc|c} 0 & 1 & 0 \\ 1 & -1 & 1 \\ \hline 1 & 0 & 0 \end{array} \right]$?

$\|G\|_\infty =$ _____, because

Part III

State feedback techniques

Question 45

Consider $\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$. Find the state-feedback gain K assigning all closed-loop poles to -1 .

$K = \begin{bmatrix} & \end{bmatrix}$, because

Question 46

Consider $x[t + 1] = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x[t] + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u[t]$. Find the state-feedback gain K rendering the closed-loop dynamics deadbeat.

$K = \begin{bmatrix} & \end{bmatrix}$, because _____

Question 47

Can $T_{yv}(s) = 1/(s+1)^3$ be the transfer function of the closed-loop system $v \mapsto y$ for a stabilizing state feedback $u = Kx + v$ for $P(s) = (s+2)/s^4$?

yes/no, because _____

Question 48

Can $T_{yv}(s) = 1/(s+1)^3$ be the transfer function of the closed-loop system $v \mapsto y$ for a stabilizing state feedback $u = Kx + v$ for $P(s) = (s-2)/s^4$?

yes/no, because _____

Question 49

Can $T_{yv}(s) = 1/(s+1)^4$ be the transfer function of the closed-loop system $v \mapsto y$ for a stabilizing state feedback $u = Kx + v$ for $P(s) = (s+2)/s^4$?

yes/no, because _____

Question 50

Can $V(x) = x' \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} x$ be a Lyapunov function candidate?

yes/no, because _____

Question 51

Does the asymptotic stability of $\dot{x}(t) = Ax(t) + Bu(t)$ imply that $\frac{d}{dt} \|x(t)\|^2 \leq 0$ for all $t \geq 0$? No need to explain.

yes/no

Question 52

Does the asymptotic stability of $\dot{x}(t) = Ax(t) + Bu(t)$ imply that there is $P > 0$ such that $\frac{d}{dt} \|P^{1/2}x(t)\|^2 \leq 0$ for all $t \geq 0$?

yes/no, because _____

Question 53

Write the task of finding $X > 0$ such that $A'X + XA + X^2 < 0$ as an LMI in some decision variable(s), no need to explain.

Question 54

Write the task of finding $Y > 0$ and L such that $(A + LC)Y + Y(A + LC)' + Y^2 < 0$ as an LMI in some decision variable(s).

_____, because _____

Question 55

Is $\bar{X} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ the stabilizing solution to the CARE $\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \bar{X} + \bar{X} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} - \bar{X} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \bar{X} = 0$?

yes/no, because _____

Question 56

Does the CARE $\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \bar{X} + \bar{X} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} - \bar{X} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \bar{X} = 0$ have a stabilizing solution?

yes/no, because _____

Question 57

Does the DARE $\bar{X} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \bar{X} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \bar{X} \begin{bmatrix} 1 & 0 \end{bmatrix} (1 + \begin{bmatrix} 1 & 0 \end{bmatrix} \bar{X} \begin{bmatrix} 1 & 0 \end{bmatrix})^{-1} \begin{bmatrix} 1 & 0 \end{bmatrix} \bar{X} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ have a stabilizing solution?

yes/no, because _____

Question 58

Can a Hamiltonian matrix $H \in \mathbb{R}^{4 \times 4}$ have $\text{spec}(H) = \{-1 \pm j, \pm j\}$?

yes/no, because _____

Question 59

Can a Hamiltonian matrix $H \in \mathbb{R}^{6 \times 6}$ have $\text{spec}(H) = \{-1 \pm j, 0, 0, 1 \pm j\}$?

yes/no, because _____

Question 60

Given $\dot{x}(t) = Ax(t) + Bu(t)$, what LQR weights Q , S , and R yield the cost function $\int_0^\infty (y^2(t) + 2u^2(t))dt$ (here $y = Cx$)?

$Q =$ _____, $S =$ _____, and $R =$ _____

Question 61

Given $\dot{x}(t) = Ax(t) + Bu(t)$, what LQR weights Q , S , and R yield the cost function $\int_0^\infty (0.2y^2(t) + 0.8\dot{y}^2(t) + u^2(t))dt$ (here $y = Cx$)?

$Q =$ _____, $S =$ _____, and $R =$ _____

Question 62

Given $x[t+1] = Ax[t] + Bu[t]$, what LQR weights Q , S , and R yield the cost function $\sum_{t=0}^\infty (y[t+1] - y[t])^2$ (here $y = Cx$)?

$Q =$ _____, $S =$ _____, and $R =$ _____

Question 63

Write a quadratic cost function, whose minimization for an LTI plant ensures that closed-loop poles are in $\{s \mid \operatorname{Re} s < -2\}$.

$\mathcal{J} = \int_0^\infty$ _____ dt , because _____

Question 64

Prove that if $\mathcal{A}_{1-3}$ hold, then the optimal continuous-time LQR controller is $u = -R^{-1}(S' + B'\bar{X})x$, where $\bar{X} \geq 0$ is the stabilizing solution to the corresponding CARE.

Question 65

Prove that if $\mathcal{A}_{1-3}$ hold, then the optimal discrete-time LQR controller is $u = -(R + B'\bar{X}B)^{-1}(S' + B'\bar{X}A)x$, where $\bar{X} \geq 0$ is the stabilizing solution to the corresponding DARE.

Question 66

Prove that the LQR controller with $m = 1$ and $S = 0$ guarantees a phase margin of at least 60° .

Question 67

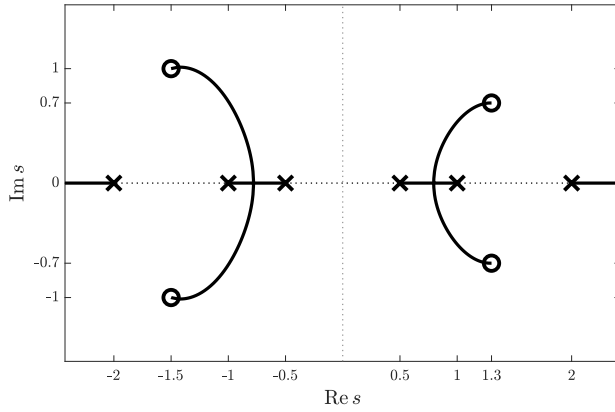
The loop transfer function for the state feedback $u = Kx$ is $L(s) = -K(sI - A)^{-1}B$. Can $L(s) = (s-1)/(s+1)^2$ be the loop transfer function designed by LQR?

yes/no, because _____

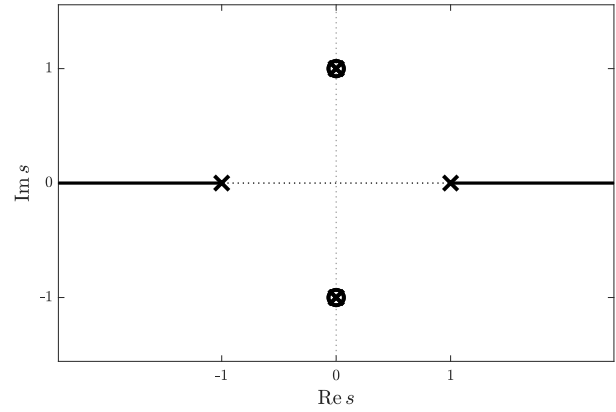
Question 68

The loop transfer function for the state feedback $u = Kx$ is $L(s) = -K(sI - A)^{-1}B$. Can $L(s) = 1/(s+1)^2$ be the loop transfer function designed by LQR with $S = 0$?

yes/no, because _____



(a) for Question 71



(b) for Question 72

Figure 1: Root-locus plots

Question 69

Let $P(s) = 1/(s+1)^2$. Write its state-space realization and design the state feedback law $u = Kx$ minimizing $\int_0^\infty u^2(t)dt$.

$K = \begin{bmatrix} \end{bmatrix}$, because

Question 70

Let $P(s) = 1/(s^2 - a^2)$ for $a > 0$. Write its state-space realization and design the state feedback law $u = Kx$ minimizing $\int_0^\infty u^2(t)dt$.

$K = \begin{bmatrix} \end{bmatrix}$, because

Question 71

Can the root locus in Fig. 1(a) belong to $1 + \frac{1}{r}B'(-sI - A')^{-1}Q(sI - A)^{-1}B = 0$ under $r > 0$ and $Q \geq 0$?

yes/no, because

Question 72*

The root locus in Fig. 1(b) corresponds to $1 + \frac{1}{r}B'(-sI - A')^{-1}Q(sI - A)^{-1}B = 0$ under controllable $(A, B) \in \mathbb{R}^{2 \times 2} \times \mathbb{R}^{2 \times 1}$, $r > 0$, and $Q \geq 0$. Which one of assumptions $\mathcal{A}_{1-3}$ is not satisfied in this case?

$\mathcal{A}_{}$, because

Part IV

Observers and observer-based feedback

Question 73

Consider $\begin{cases} \dot{x}(t) = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(t) \\ y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t) \end{cases}$. Find the observer gain L assigning all poles of the full-order observer to -1 .

$L = \begin{bmatrix} \\ \end{bmatrix}$, because _____

Question 74

Consider the system

$$\begin{cases} \dot{x}(t) = Ax(t), & x(0) = x_0 \\ y(t) = Cx(t) + Du(t) \end{cases}$$

Suggest an observer for its state having the property that the estimation error $\epsilon = x - \hat{x}$ is independent of the control signal u and the convergence of the error can be affected by the choice of the gain L , provided (C, A) is observable. Prove that via presenting the dynamic equation for ϵ .

Question 75

Suppose $\text{spec}(A + BK) = \{-1, -2\}$ and $\text{spec}(A + LC) = \{-3, -4\}$. What is the spectrum of $\begin{bmatrix} A & BK \\ -LC & A + BK + LC \end{bmatrix}$?

$\text{spec}\left(\begin{bmatrix} A & BK \\ -LC & A + BK + LC \end{bmatrix}\right) = \{ \}$, because _____

Question 76

Consider two observer-based controllers $u = K\hat{x} + v$ for the same stabilizing K . The first chooses the observer poles in $\{s \in \mathbb{C} \mid \text{Re } s < -1\}$ and the second in $\{s \in \mathbb{C} \mid \text{Re } s < -10\}$. Both controllers know the initial conditions of the plant. Which controller results in a faster response to v ?

The first / second, because _____

Question 77

What class of signals can be generated by the exosystem $\begin{cases} \dot{x}(t) = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} x(t) \\ y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t) \end{cases}$ started from arbitrary initial conditions?

$y(t) = $, because _____

Question 78

What class of signals can be generated by the exosystem $\begin{cases} x[t+1] = x[t] \\ y[t] = x[t] \end{cases}$ started from arbitrary initial conditions?

$y[t] = $, because _____

Formulae¹

Matrix equations

- Sylvester: $A_1 X + X A_2 + Q = 0$
- Lyapunov: $A X + X A' + Q = 0$
- discrete Lyapunov: $X = A X A' + Q$
- continuous-time algebraic Riccati: $A' \bar{X} + \bar{X} A + Q - (S + \bar{X} B) R^{-1} (S' + B' \bar{X}) = 0$
- discrete-time algebraic Riccati: $\bar{X} = A' \bar{X} A + Q - (S + A' \bar{X} B) (R + B' \bar{X} B)^{-1} (S' + B' \bar{X} A)$

Other

- Ackermann: $K = -[0 \ \cdots \ 0 \ 1] M_c^{-1} \chi_{cl}(A)$
- continuous-time LQR solvability conditions:
 - $\mathcal{A}_1$: (A, B) is stabilizable
 - $\mathcal{A}_2$: $R > 0$ and $Q - S R^{-1} S' \geq 0$
 - $\mathcal{A}_3$: $\begin{bmatrix} A - j\omega I & B \\ Q & S \\ S' & R \end{bmatrix}$ has full column rank for all $\omega \in \mathbb{R}$
- (continuous-time) Kalman–Bucy solvability conditions:
 - $\mathcal{A}_4$: (C, A) is detectable
 - $\mathcal{A}_5$: $D_w D_w' > 0$
 - $\mathcal{A}_6$: $\begin{bmatrix} A - j\omega I & B_w \\ C & D_w \end{bmatrix}$ has full row rank for all $\omega \in \mathbb{R}$
- discrete-time LQR solvability conditions:
 - $\mathcal{A}_1$: (A, B) is stabilizable
 - $\mathcal{A}_2$: $\begin{bmatrix} Q & S \\ S' & R \end{bmatrix} \geq 0$
 - $\mathcal{A}_3$: $\begin{bmatrix} A - e^{j\theta} I & B \\ Q & S \\ S' & R \end{bmatrix}$ has full column rank for all $\theta \in [-\pi, \pi]$
- (discrete-time) Kalman solvability conditions:
 - $\mathcal{A}_4$: (C, A) is detectable
 - $\mathcal{A}_5$: $\begin{bmatrix} A - e^{j\theta} I & B_w \\ C & D_w \end{bmatrix}$ has full row rank for all $\theta \in [-\pi, \pi]$
- key continuous-time LQR relation: $\begin{bmatrix} Q & S \\ S' & R \end{bmatrix} = \begin{bmatrix} -K' \\ I \end{bmatrix} R \begin{bmatrix} -K & I \end{bmatrix} - \begin{bmatrix} \bar{X} \\ 0 \end{bmatrix} \begin{bmatrix} A & B \end{bmatrix} - \begin{bmatrix} A' \\ B' \end{bmatrix} \begin{bmatrix} \bar{X} & 0 \end{bmatrix}$
- continuous-time return-difference equality: $(1 + L_{sf}(-s))(1 + L_{sf}(s)) = 1 + \frac{1}{r} B'(-sI - A')^{-1} Q(sI - A)^{-1} B$ (provided $m = 1$ and $S = 0$)
- key discrete-time LQR relation: $\begin{bmatrix} Q & S \\ S' & R \end{bmatrix} = \begin{bmatrix} -K' \\ I \end{bmatrix} (R + B' \bar{X} B) \begin{bmatrix} -K & I \end{bmatrix} + \begin{bmatrix} \bar{X} & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} A' \\ B' \end{bmatrix} \bar{X} \begin{bmatrix} A & B \end{bmatrix}$
- regulator equation: $\begin{bmatrix} A & B \\ C_e & 0 \end{bmatrix} \begin{bmatrix} \Pi \\ \Gamma \end{bmatrix} - \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Pi \\ \Gamma \end{bmatrix} A_{ex} + \begin{bmatrix} B_d C_{ex} \\ 0 \end{bmatrix} = 0$

¹Don't pavlov on their use.