



STATE-SPACE CONTROL FUNDAMENTALS (00860721)

HOMEWORK 4

(submission deadline: 24/7/2026, 13:00; do make an effort to be concise, clear, and accurate)

Question 1. A linearized model of a Furuta pendulum in the “down” position, like that in Fig. 1, is

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & J_b + mr^2 & 0 & mrl \\ 0 & 0 & 1 & 0 \\ 0 & mrl & 0 & J_p + ml^2 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -f & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -gml & -\zeta \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \tau_m, \quad (1)$$

where

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} := \begin{bmatrix} \phi \\ \dot{\phi} \\ \theta \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \text{arm angle} \\ \text{arm angular velocity} \\ \text{pendulum angle} \\ \text{pendulum angular velocity} \end{bmatrix}$$

(the angles are in radians). All physical parameters are like in Homework 2. The torque τ_m applied to the arm is supposed to be an *unmeasured* disturbance. Assuming numerical values of the system parameters as

Parameter	l	r	m	J_p	ζ	J_b	f	g
Value (in SI)	0.3226	0.3	0.04	0.0003	0.0005	0.01	0.01	9.81

Table 1: Parameters of the system, same as in Homework 2

in Table 1, construct the A and B_d matrices of the realization. As a debugging hint, the poles of the system for the parameters in Table 1 should be in $\{-0.7387, -0.1932 \pm j6.1198, 0\}$. In all simulations below take $\tau_m(t) = 0.1 \sin(\pi t)$ and the time axis $[0, 9.2]$.

1. Assume that only the pendulum angle θ can be measured, with some noise n_θ . Formulate the least-square estimation problem assuming $w = (\tau_m, n_\theta)$, i.e. form the matrices A , B_w , D_w (via the intensity of n_θ), and C . By tuning D_w , design a Kalman–Bucy filter ensuring that the estimation error

$$\epsilon = \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \end{bmatrix} := \begin{bmatrix} \phi - \hat{\phi} \\ \theta - \hat{\theta} \end{bmatrix}$$

satisfies $|\epsilon_i(t)| \leq 0.1$ for all $t \geq 0$ and $i \in \mathbb{Z}_{1..2}$ under $n_\theta = 0$. Is it possible? If not, explain why. If yes, also explain why and present three plots: one with ϕ and its estimate $\hat{\phi}$, one with θ and $\hat{\theta}$, and the last one with ϵ (for the ordinate range $[-0.105, 0.105]$).

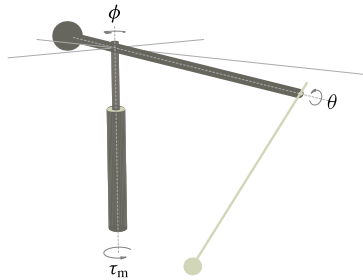


Fig. 1: Furuta pendulum.

2. Now assume that only the arm pendulum angle ϕ can be measured, with some noise n_ϕ . Formulate the least-square estimation problem assuming $w = (\tau_m, n_\phi)$, i.e. form the matrices A , B_w , D_w (via the intensity of n_ϕ), and C . By tuning D_w , design a Kalman–Bucy filter ensuring that the estimation error ϵ satisfies $|\epsilon_i(t)| \leq 0.1$ for all $t \geq 0$ and $i \in \mathbb{Z}_{1..2}$ under $n_\phi = 0$. Is it possible? If not, explain why. If yes, also explain why and present three plots: one with ϕ and its estimate $\hat{\phi}$, one with θ and $\hat{\theta}$, and the last one with ϵ (for the ordinate range $[-0.105, 0.105]$).
3. Simulate the results of the previous item under $n_\phi(t) = 10 \cos(10\pi t)$, presenting the same 3 plots as there (except the ordinate range of the third plot). Explain differences.
4. Finally, assume that both ϕ and θ are measurable, each with an independent noise, n_ϕ and n_θ . Formulate the least-square estimation problem assuming $w = (\tau_m, n_\phi, n_\theta)$, i.e. form the matrices A , B_w , D_w (via the intensities of n_ϕ and n_θ), and C . Assuming that the measurement of θ is of higher quality than that of ϕ , tune D_w to end up with a Kalman–Bucy filter ensuring that the estimation error ϵ satisfies $|\epsilon_i(t)| \leq 0.1$ for all $t \geq 0$ and $i \in \mathbb{Z}_{1..2}$ under $n_\phi(t) = 10 \cos(10\pi t)$ and $n_\theta = 0$. Present three plots: one with ϕ and its estimate $\hat{\phi}$, one with θ and $\hat{\theta}$, and the last one with ϵ (for the ordinate range $[-0.105, 0.105]$, again). Explain your choice of intensities of n_ϕ and n_θ and the resulting behaviors of the components of ϵ .