



STATE-SPACE CONTROL FUNDAMENTALS (00860721)

HOMEWORK 3

(submission deadline: 8/7/2026, 13:00; do make an effort to be concise, clear, and accurate)

Question 1. Consider the same inverted Furuta pendulum as in HW2, with the same state-space realization and the same parameters. Mind that the input to the system is the DC motor voltage.

- Design the minimum-energy control signal u steering the state of the system from $x(0) = 0$ to

$$x(t_f) = \begin{bmatrix} \phi(t_f) \\ \dot{\phi}(t_f) \\ \theta(t_f) \\ \dot{\theta}(t_f) \end{bmatrix} = \begin{bmatrix} \pi/2 \\ 0 \\ 0 \\ 0 \end{bmatrix} =: x_f, \quad \text{where } x = \begin{bmatrix} \text{arm angle} \\ \text{arm angular velocity} \\ \text{pendulum angle} \\ \text{pendulum angular velocity} \end{bmatrix}$$

for a given t_f and keeping it there for $t > t_f$. What is u_f required to have x_f above in the equilibrium? Choose t_f as the smallest time instance such that $|u(t)| \leq 1$ for all $t \in [0, t_f]$. Plot u and the resulting x (on 5 separate plots, with angles in degrees) for $t \in [0, 1.3t_f]$. Make sure that $u(t) = u_f$ for all $t > t_f$. Explain the waveform of the resulting ϕ intuitively, as somebody knowing nothing about controls, but still trying to move a palm balancing a rod on it to a different position.

Hint: Simulating this open-loop system might be sensitive to roundoff errors, so select a sufficiently dense time grid. However, a dense grid could inflate plot sizes. So take something like `t = linspace(0, 1.3*t_f, round(130*Nt*t_f)+1);` for a sufficiently large integer Nt and then plot it like `plot(t(1:Nt:end), ureq(1:Nt:end))`.

- Assume now that there is a disturbance torque τ_d acting on the rotating base, in addition to τ_m . Derive its “ B_d ” parameter analytically, in the spirit of modeling in HW2. How is this disturbance τ_d classified (according to the terminology of Lecture 3, Slide 2)?
- Simulate the system in Item 1, also for $t \in [0, 1.3t_f]$, now with $\tau_d(t) = 0.1\mathbb{1}(t - 1.1t_f)$, in the same interval like before. Present 2 plots, those of ϕ and θ (in degrees, as before). Explain the result.
- Use the state-feedback gain K designed in HW2 to implement the 2DOF state-feedback tracking setup. Simulate the response of the closed-loop system to the reference signals y_{ref} and u_{req} designed as in Item 1, still for $t \in [0, 1.3t_f]$, and the disturbance τ_d satisfying $\tau_d(t) = 0.1\mathbb{1}(t)$. Present 2 plots, one showing ϕ_{ref} and ϕ and another one showing θ_{ref} and θ (in degrees, as before). Explain differences from the simulation in Item 3. Calculate (not from the plots) the bias of the steady state values of ϕ and θ with respect to those of ϕ_{ref} and θ_{ref} , respectively. Why does the disturbance affect those values differently for ϕ and θ (explain by systems theory arguments)?
- Suggest (and justify) how to redesign the state-feedback gain K (still by the LQR technique) to reduce the steady-state error $\lim_{t \rightarrow \infty} |\phi(t) - \phi_{\text{ref}}(t)|$ by a factor of 2 with respect to that in Item 4. Simulate the redesigned system and present the same two plots as in Item 4.

Submit a self-contained M-file with all simulations. To separate simulations for different items, define the variable `item` and organize simulations via the “switch item” environment. In other words, make sure that I could change the value of one variable, `item`, to simulate only one corresponding item. Also, please comment out all `exportgraphics` (or other commands producing plot files).