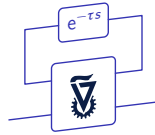


Time Delays in Control and Estimation (038806)



lecture no. 1

Leonid Mirkin

Faculty of Mechanical Engineering
Technion—IIT

1/24

Outline

Course info

Time-delay systems in control applications

Background requirements

2/24

General info

- Credit points: **3**
- Prerequisite: any **advanced linear control** course preferably, as in <http://leo.technion.ac.il/Courses/LCS/LCSnotes19.pdf>
- Grading policy: homework **100%** (4 best out of 5 assignments)
Homework solutions must be submitted **electronically** to c038806@technion.ac.il
- Course site: <http://leo.technion.ac.il/Courses/TDCE/>
- Literature:
 1. My **lecture notes** (available from the site).
 2. J. E. Marshall, H. Górecki, A. Korytowski, and K. Walton, *Time-Delay Systems: Stability and Performance Criteria with Applications*. London: Ellis Horwood, 1992.
 3. K. Gu, V. L. Kharitonov, and J. Chen, *Stability of Time-Delay Systems*. Boston: Birkhäuser, 2003.
 4. R. F. Curtain and H. Zwart, *An Introduction to Infinite-Dimensional Linear Systems Theory*. New York: Springer-Verlag, 1995.

3/24

Syllabus

1. **Introduction**
 - time delays in engineering applications; system-theoretic preliminaries
2. **Mathematical modeling of time-delay systems**
 - frequency domain & modal analyses; state space; rational approximations
3. **Stability analysis**
 - modal analyses; Lyapunov's method
4. **Stabilization methods**
 - fixed-structure controllers; Smith, DTC & C° ; FSA; coprime factorization
5. **Handling uncertain delays**
 - unstructured uncertainty embedding; Lyapunov-based methods
6. **Performance**
 - tuning industrial controllers
 - H_2 and H_∞ optimizations
7. **Implementation of dead-time compensation**
8. **Systems with preview**
 - preview tracking and smoothing
9. **Exploiting delays in control**
 - repetitive control; input shaping; PID; weird loop shaping; . . .

4/24

On a less formal side

What this course is about...

- system-theoretic and control aspects of delays in dynamical systems
- exploiting the structure of the delay element
- giving a flavor of ideas in the field
- showing that things are (relatively) simple if the viewpoint is right

...and what it isn't

- digging up most general and mathematically intriguing cases
- answering the very problem that motivated you to take this course

5/24

Outline

Course info

Time-delay systems in control applications

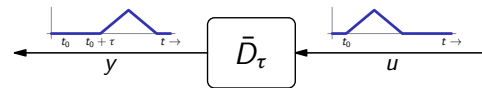
Background requirements

6/24

Time delay element

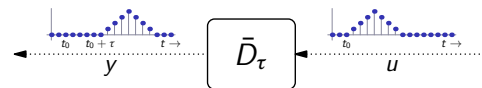
Given $\tau \in \mathbb{R}_+$,

$$y = \bar{D}_\tau u : \quad y(t) = u(t - \tau)$$



or given $\tau \in \mathbb{N}$,

$$y = \bar{D}_\tau u : \quad y[t] = u[t - \tau]$$



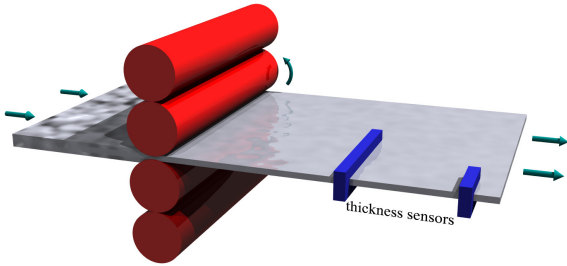
7/24

Why delays?

- Delays in physical processes
 - loop delays
 - process delays
 - ...
- Compact/economical approximations of complex dynamics
- Exploiting delays to improve performance

8/24

Loop delays: steel rolling

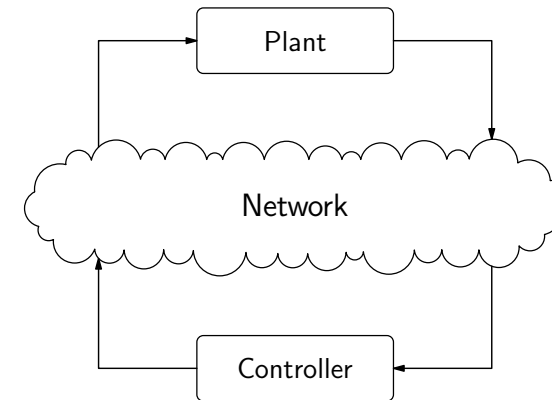


Thickness can only be measured at some distance from rolls, leading to

- measurement delays

9/24

Loop delays: networked control

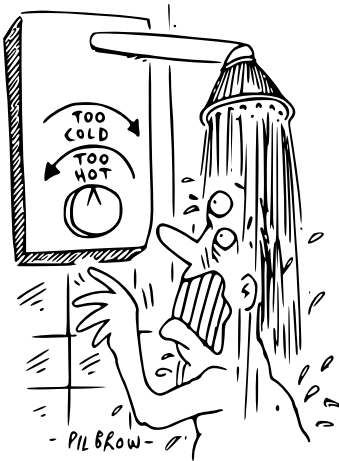


Sampling, encoding, transmission, decoding need time. This gives rise to

- measurement delays
- actuation delays

10/24

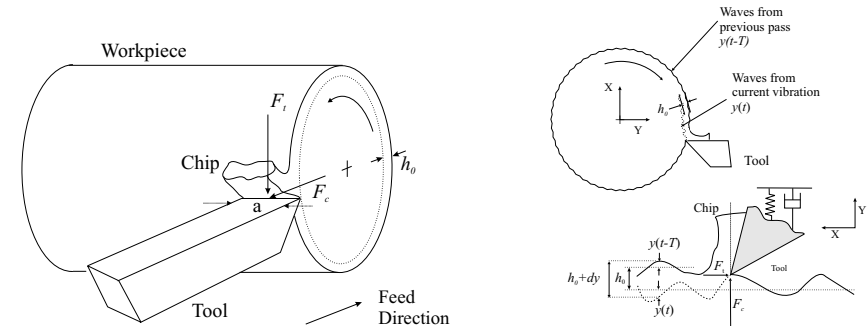
Loop delays: temperature control



Everybody experienced this, I guess...

11/24

Internal delays: regenerative chatter in metal cutting



Deviation from the ideal cut satisfies

$$m\ddot{y}(t) + c\dot{y}(t) + ky(t) = K_f a (h_0 + y(t - T) - y(t)),$$

where T is time of one full rotation of workpiece.

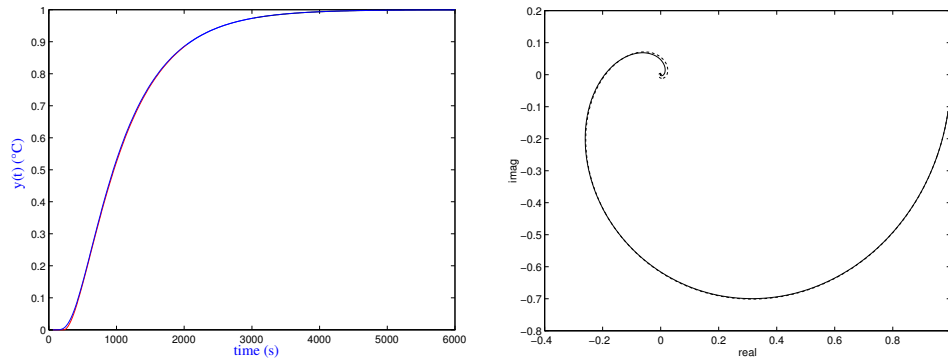
12/24

Delays as modeling tool: heating a can

Transfer function of a heated can (derived from PDE model):

$$P(s) = \frac{1}{J_0(\sqrt{\frac{-s}{\alpha}}R)} + \sum_{m=1}^{\infty} \frac{2}{\lambda_m R J_1(\lambda_m R)} \frac{s}{s + \alpha \lambda_m^2} \frac{1}{\cosh(\sqrt{\frac{s}{\alpha}} + \lambda_m^2 \frac{L}{2})}$$

Its approximation by $P_2(s) = \frac{1}{(\tau_1 s + 1)(\tau_2 s + 1)} e^{-\tau s}$ is reasonably accurate:



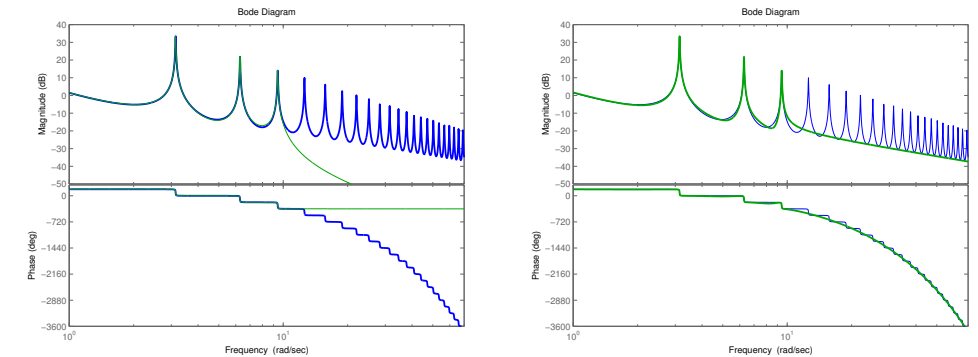
13/24

Delays as modeling tool: torsion of a rod

Transfer function of a free-free uniform rod at distance x from actuator:

$$P(s) = \frac{k}{s} \frac{e^{-x\tau s} + e^{-(2-x)\tau s}}{1 - e^{-2\tau s}}, \quad 0 \leq x \leq 1.$$

Its approximation by $P_r(s)e^{-x\tau s}$ does capture high-frequency phase lag:



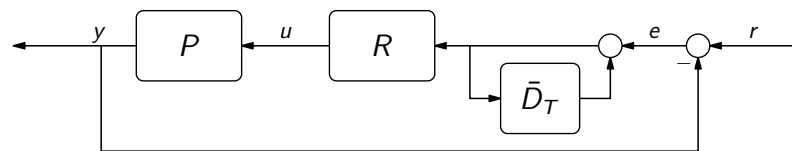
14/24

Exploiting delays: repetitive control

Any T -periodic signal f satisfies

$$f(t) - f(t - T) = 0 \quad \Longleftrightarrow \quad F(s) = \frac{1}{1 - e^{-sT}}$$

This motivates the configuration, called **repetitive control**:



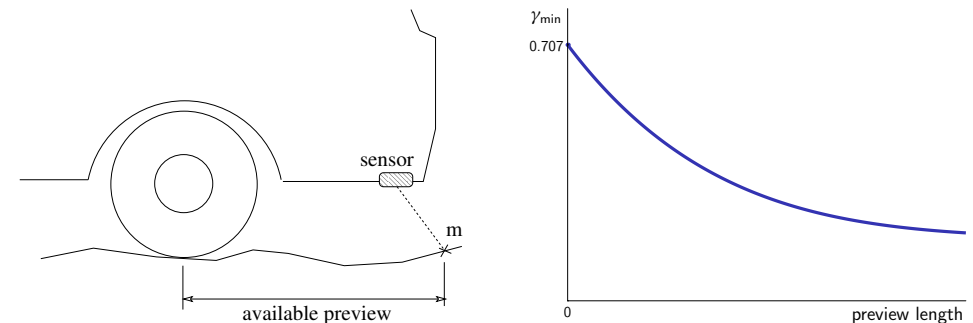
which is

- generalization of the internal model controllers (including I)
- and guarantees (if stable!)
- asymptotically **perfect tracking** of **any T -periodic reference r**

15/24

Exploiting delays: preview control

If we know reference in advance, we can exploit it to improve performance:



16/24

Outline

Course info

Time-delay systems in control applications

Background requirements

17/24

Notation

$\mathbb{R}$	the set of real numbers, $\mathbb{R} = (-\infty, \infty)$
$\mathbb{R}_+$	the set of nonnegative real numbers, $\mathbb{R}_+ = [0, \infty)$
$\mathbb{R}_-$	the set of nonpositive real numbers, $\mathbb{R}_- = (-\infty, 0]$
$j\mathbb{R}$	the set of pure imaginary numbers
$\mathbb{C}$	the set of complex numbers
$\mathbb{C}_\alpha$	the half plane to the right of $\alpha \in \mathbb{R}$, i.e. $\mathbb{C}_\alpha = \{z \in \mathbb{C} \mid \operatorname{Re} z > \alpha\}$
$\bar{\mathbb{C}}_\alpha$	the closure of $\mathbb{C}_\alpha$, i.e. $\bar{\mathbb{C}}_\alpha = \{z \in \mathbb{C} \mid \operatorname{Re} z \geq \alpha\}$
$\mathbb{T}$	the unit circle, $\mathbb{T} := \{z \in \mathbb{C} \mid z = 1\}$
$\mathbb{F}$	alias of either $\mathbb{R}$ or $\mathbb{C}$
$\mathbb{Z}$	the set of integers
$\mathbb{N}$	the set of positive integers (natural numbers)
$\mathbb{Z}_+$	the set of nonnegative integers
$\mathbb{Z}_-$	the set of nonpositive integers, $\mathbb{Z}_- = \mathbb{Z} \setminus \mathbb{N}$
$\mathbb{Z}_{t_1..t_2}$	the interval $\{t_1, t_1 + 1, \dots, t_2\}$

18/24

Notation (contd)

$\mathbb{1}_{\mathbb{I}}(t)$	(continuous-time) characteristic function of a set $\mathbb{I} \subset \mathbb{R}$ $\mathbb{1}_{\mathbb{I}}(t) = \begin{cases} 1 & \text{if } t \in \mathbb{I} \\ 0 & \text{otherwise} \end{cases}$
$\mathbb{1}(t)$	the unit step, $\mathbb{1}(t) := \mathbb{1}_{\mathbb{R}_+}(t)$
$\delta(t)$	the Dirac delta function
$\mathbb{1}_{\mathbb{I}}[t]$	(discrete-time) characteristic function of a set $\mathbb{I} \subset \mathbb{Z}$ $\mathbb{1}_{\mathbb{I}}[t] = \begin{cases} 1 & \text{if } t \in \mathbb{I} \\ 0 & \text{otherwise} \end{cases}$
$\mathbb{1}[t]$	the unit step, $\mathbb{1}[t] := \mathbb{1}_{\mathbb{Z}_+}[t]$
$\delta[t]$	the unit pulse at $i = 0$, $\delta[t] = \begin{cases} 1 & \text{if } t = 0 \\ 0 & \text{otherwise} \end{cases}$
L_2/ℓ_2	space of square integrable/summable functions
H_∞	space of functions $\mathbb{C}_0 \rightarrow \mathbb{C}$ holomorphic and bounded in $\mathbb{C}_0$

19/24

Notation (contd)

e_i	the i th standard basis in $\mathbb{F}^n$, i.e. $e_1 := \begin{bmatrix} 1 \\ 0 \\ \vdots \end{bmatrix}$, $e_2 := \begin{bmatrix} 0 \\ 1 \\ \vdots \end{bmatrix}$, etc.
I_n	$n \times n$ identity matrix (or simply I)
M'	(complex-conjugate) transpose of a matrix $M \in \mathbb{F}^{p \times m}$
$\lambda_i(M)$	i th eigenvalue of $M \in \mathbb{F}^{n \times n}$
$\operatorname{spec}(M)$	spectrum of $M \in \mathbb{F}^{n \times n}$ (the set of all its eigenvalues)
$\rho(M)$	spectral radius of $M \in \mathbb{F}^{n \times n}$, $\rho(M) = \max_i \{ \lambda_i(M) \}$
$\bar{\sigma}(M)$	the maximal singular value of $M \in \mathbb{F}^{p \times m}$
$\underline{\sigma}(M)$	the minimal singular value of $M \in \mathbb{F}^{p \times m}$
$\operatorname{tr}(M)$	trace of $M \in \mathbb{F}^{n \times n}$, $\operatorname{tr}(M) = \sum_{i=1}^n m_{ii} = \sum_{i=1}^n \lambda_i(M)$
$\ M\ $	spectral norm of $M \in \mathbb{F}^{n \times m}$, $\ M\ ^2 := \rho(M'M) = \rho(MM')$
$\ M\ _F$	Frobenius norm of $M \in \mathbb{F}^{n \times m}$, $\ M\ _F^2 := \operatorname{tr}(M'M) = \operatorname{tr}(MM')$

20/24

Continuous-time linear systems facts

If causal system $G : u \mapsto y$ is linear time-invariant (LTI),

- convolution representation

$$y(t) = \int_0^t g(t-s)u(s)ds, \text{ where } g(t) \text{ is the impulse response of } G$$

- transfer function $G(s) = \mathcal{L}\{g(t)\}$, so that

$$Y(s) = G(s)U(s)$$

- state-space representation (if G is finite dimensional)

$$G : \begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) + Du(t) \end{cases}$$

with $g(t) = D\delta(t) + Ce^{At}B$ & $G(s) = D + C(sI - A)^{-1}B$. The **state vector** x is **history accumulator**, i.e. if we know $x(t_0)$, no knowledge of $u(t)$ in $t < t_0$ is required to calculate $y(t)$ for $t \geq t_0$.

21/24

Discrete-time linear systems facts

If causal system $G : u \mapsto y$ is linear shift-invariant (LSI),

- convolution representation

$$y[t] = \sum_{s=0}^t g[t-s]u[s], \text{ where } g[t] \text{ is the impulse response of } G$$

- transfer function $G(z) = \mathcal{Z}\{g[t]\}$, so that

$$Y(z) = G(z)U(z)$$

- state-space representation (if G is finite dimensional)

$$G : \begin{cases} x[t+1] = Ax[t] + Bu[t] \\ y[t] = Cx[t] + Du[t] \end{cases}$$

with $g[t] = D\delta[t] + CA^tB$ and $G(z) = D + C(zI - A)^{-1}B$. The **state vector** x is **history accumulator**, i.e. if we know $x[t_0]$, no knowledge of $u[t]$ in $t < t_0$ is required to calculate $y[t]$ for $t \geq t_0$.

22/24

Manipulations via realizations

Denote

$$\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array} \right] := \begin{cases} D + C(sI - A)^{-1}B \\ D + C(zI - A)^{-1}B \end{cases}$$

- Addition (parallel interconnection):

$$\left[\begin{array}{c|c} A_1 & B_1 \\ \hline C_2 & D_1 \end{array} \right] + \left[\begin{array}{c|c} A_2 & B_2 \\ \hline C_2 & D_2 \end{array} \right] = \left[\begin{array}{cc|c} A_1 & 0 & B_1 \\ 0 & A_2 & B_2 \\ \hline C_1 & C_2 & D_1 + D_2 \end{array} \right]$$

- Multiplication (series interconnection):

$$\left[\begin{array}{c|c} A_2 & B_2 \\ \hline C_2 & D_2 \end{array} \right] \left[\begin{array}{c|c} A_1 & B_1 \\ \hline C_1 & D_1 \end{array} \right] = \left[\begin{array}{cc|c} A_1 & 0 & B_1 \\ B_2C_1 & A_2 & B_2D_1 \\ \hline D_2C_1 & C_2 & D_2D_1 \end{array} \right] = \left[\begin{array}{cc|c} A_2 & B_2C_1 & B_2D_1 \\ 0 & A_1 & B_1 \\ \hline C_2 & D_2C_1 & D_2D_1 \end{array} \right]$$

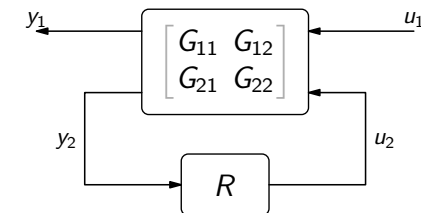
- Inverse (exists in this form iff $\det D \neq 0$):

$$\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array} \right]^{-1} = \left[\begin{array}{c|c} A - BD^{-1}C & BD^{-1} \\ \hline -D^{-1}C & D^{-1} \end{array} \right]$$

23/24

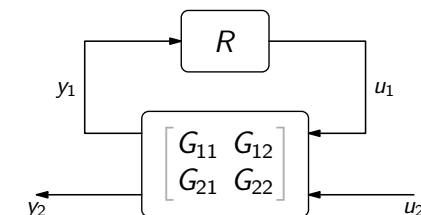
Linear-fractional transformations

Lower LFT:



$$\mathcal{F}_l(G, R) = G_{11} + G_{12}R(I - G_{22}R)^{-1}G_{21}$$

Upper LFT:



$$\mathcal{F}_u(G, R) = G_{22} + G_{21}R(I - G_{11}R)^{-1}G_{12}$$

24/24