



TIME DELAYS IN CONTROL AND ESTIMATION (038806)

HOMEWORK 3

(submission deadline: 20/8/2024, 23:59; do make an effort to be concise, clear, and accurate*)

Problem 1 (50%). Consider the system (which is motivated by the dynamics of machining chatter):

$$\dot{x}(t) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -10-k & 10 & 0 & 0 \\ 5 & -15 & 0 & -0.25 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ k & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} x(t-\tau),$$

for some $k \in \mathbb{R}$.

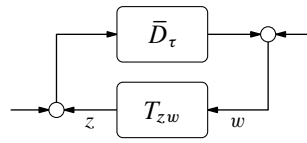


Fig. 1: Delay margin setup

1. Present the system in the form of Fig. 1 with scalar z and w .
2. Calculate the actual, nonconservative, delay margins for $k \in \{0.2, 2\}$.
3. Calculate lower bounds on the delay margin using the Small Gain Theorem for $k \in \{0.2, 2\}$. Try to maximize these lower bounds (in any case, the lower bound should not be lower than $2/3$ of the precise delay margin).

Remark: Check yourselves carefully, it's easy to end up with a wrong result because of code errors. As a reference point, the small-gain analysis with $W_x(s) = 1$ and $W_\alpha(s) = \bar{\tau}s$ shall result in $\bar{\tau} \approx 0.0624$.

Problem 2 (50%). Let

$$\dot{x}(t) = \begin{bmatrix} 1 & 1 \\ 0 & -2 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} x(t-1) + \begin{bmatrix} -2 \\ 1 \end{bmatrix} u(t-1) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} d(t)$$

be the plant, whose measured variables are x_1 and x_2 , control input is u , and d is a disturbance. The control law

$$u(t) = 2e \left(\frac{3 - e^{-1}}{5 - 2e^{-1}} x_1(t) + \frac{1}{5 - 2e^{-1}} x_2(t) + \int_{t-1}^t e^{t-s-1} \left(\frac{1}{5 - 2e^{-1}} x_2(s) - u(s) \right) ds \right)$$

renders the characteristic function $\chi_{\tau,cl}(s) = (s+1)(s+2-e^{-s})$ and thus stabilizes the system. Implement this control law using both the reset mechanism of §7.2 and the LDA approach of §7.4 (whatever variants that work) under $d = \mathbb{1}$. Use the Simulink file hw4q2.slx available at the course site as the base for the simulation, only fill up its “Pi” block. Submit two Simulink files, for each approximation method.

*Computations may be done with whatever numerical or symbolic tools. But the logic should be explained and proofs given.