



TIME DELAYS IN CONTROL AND ESTIMATION (038806)

HOMEWORK 3

(submission deadline: 30/7/2024, 23:59; do make an effort to be concise, clear, and accurate*)

Problem 1 (50%). Consider the system

$$\dot{x}(t) = \begin{bmatrix} 1 & 1 \\ 0 & -2 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} x(t - \tau) + \begin{bmatrix} b \\ 1 \end{bmatrix} u(t - \tau)$$

where $\tau > 0$.

1. Present this system in the form of Fig. 1.7 with a 2×2 delay block $\bar{D}_\tau$ and measured x . Present G in terms of its state-space realization.
2. How many poles in $\bar{\mathbb{C}}_0$ this system has? What are them?
3. Design a stabilizing state-feedback controller that moves only unstable poles and works for every τ . What closed-loop characteristic polynomial it yields? For what b and τ is this possible?

Problem 2 (30%). Read §3.4.4 in the Lecture Notes. Use loop shifting (ad hoc, perhaps, not of the form presented in Fig. 3.11, unless you could come up with a clean analysis) to reduce the problem of stabilizing

$$P(s) = \frac{1 + R(s)e^{-\tau s}}{1 - R(s)e^{-\tau s}},$$

for some $R \in H_\infty$ to that of a finite-dimensional system. Design a stabilizing controller for P assuming that $\tau = 1$ and $R(s) = -4/(s + 1)$. Simulate its response to a step load (input) disturbance. Submit a stand-alone Simulink file with these simulations.

Problem 3 (20%). Let

$$G(s) = C(sI - A)^{-1}(B_1 e^{-\tau_1 s} + B_2 e^{-\tau_2 s})$$

for some $\tau_2 > 0$ and $\tau_1 > 0$. Suggest a system with a rational transfer function $\tilde{G}(s)$ such that

$$\sigma_\tau\{G(s)\} = \tilde{G}(s) - G(s)$$

is its FIR completion. Present an n -dimensional state-space realization of this $\tilde{G}(s)$ having the same “ A ” matrix as G .

Problem 4 (10%, bonus). Given $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$, and $\tau \in \mathbb{N}$, prove that

$$\text{rank} \begin{bmatrix} A^\tau & A^{\tau-1}B & \cdots & AB & B \end{bmatrix} = n$$

iff (A, B) has no unreachable modes at the origin.

*Computations may be done with whatever numerical or symbolic tools. But the logic should be explained and proofs given.