



TIME DELAYS IN CONTROL AND ESTIMATION (038806)

HOMEWORK 2

(submission deadline: 16/7/2024, 23:59; do make an effort to be concise, clear, and accurate*)

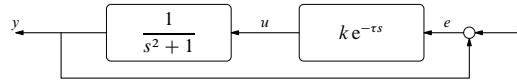


Fig. 1: Delayed control of an oscillator

Problem 1 (35%). Consider the system in Fig. 1 with some $\tau \geq 0$ (mind that the feedback is positive).

1. What is the characteristic function $\chi_\tau(s)$ of the closed-loop system?
2. For what delays τ the closed-loop system is stable under $k = 1$?
3. For what delays τ the closed-loop system is stable under $k = 0.25$?

Problem 2 (35%). Propose a single-delay quasi-polynomial $\chi_\tau(s)$ such that $\chi_0(s)$ is Hurwitz and there are exactly 4 disjoint intervals of $\tau \geq 0$ for which $\chi_\tau(s)$ is Hurwitz as well. Characterize these intervals. Present Nichols charts of $e^{-\tau s} Q_1(s)/Q_0(s)$ for 4 values of τ at the right-hand (supremal) endpoint of each of those intervals (all at one plot). Explain those charts from the Nyquist criterion viewpoint.

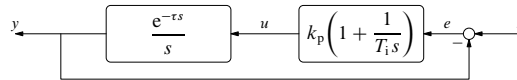


Fig. 2: PI controller for an integrator

Problem 3 (30%). Consider the control system in Fig. 2, where $\tau > 0$ and $k_p, T_i \in \mathbb{R}$. Under what conditions of k_p , T_i , and τ the closed-loop system is stable? Plot level curves in the $(k_p, 1/T_i)$ -plane for the minimal destabilizing delays $\tau \in \{1/3, 2/5, 1/2, 2/3, 1, 2\}$. What should be a strategy in terms of the parameters k_p and T_i to increase the range of delays for which the system is stable? What gain and phase margins are attained with this strategy? What crossover frequency? What is the upper bound on τ for which stabilizing k_p and T_i exist?

*Computations may be done with whatever numerical or symbolic tools. But the logic should be explained and proofs given.