



Sampled-Data Control Systems

Spring 2019/2020

Homework no. 5

(submission deadline: 23.06.2020, 11:59pm; please make an effort to be concise, clear, accurate, and rigorous)

Problem 1 (100%). Consider a pendulum on a cart having the transfer function

$$P(s) = \frac{2s^2}{(s/8 + 1)(s^2 + 0.02s + 25)}$$

This transfer function has one real pole at $s = -5$ and a pair of lightly-damped poles with the natural frequency $\omega_n = 5$ and the damping factor $\zeta = 1/500$, so that these poles are at $s = -\zeta\omega_n \pm j\sqrt{1 - \zeta^2}\omega_n$. The control goal is to dampen the pendulum against load (input) disturbances $d(t)$. Assume that the state vector, in whatever realization of P , is available at sampling instances ih , where $h > 0$ is the sampling period.

Consider first the discrete-time LQR design for the discretized plant $\bar{P}_h$ and a regulated signal z such that

$$\|z[t]\|^2 = \|\bar{y}[t + 1] - \bar{y}[t]\|^2 + \rho\|\bar{u}[t]\|^2$$

for some $\rho \geq 0$.

1. Construct C_z and D_z for any regulated signal satisfying the equality above. Keep the state vector of the discretized plant unchanged. Under what conditions on ρ assumption $\mathcal{A}_5$ is satisfied?
2. Consider $\rho = 1$ and calculate the attainable LQR performance $\gamma(h)$ for $x_0 = B$, where B is the “ B ” matrix of the chosen realization of the continuous-time P , and sampling periods taken from the following set:

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h = linspace(0, 3.5*pi/wd, 301);  
h = unique(sort([pi/wd, 2*pi/wd, 3*pi/wd, h(2:end)]));
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where $\omega_d := \sqrt{1 - \zeta^2}\omega_n$ is the damped frequency. Plot $\gamma(h)$, in dB, vs. h . Explain the results.

3. Take $h = 0.1\omega_d$ and design the optimal state feedback gain, choosing the minimal ρ under which $|u[t]| \leq 1$ for the response from $x_0 = B$. Simulate the continuous-time response of the closed-loop system under zero initial conditions and the square wave load disturbance $d(t) = \text{sign}(\sin(0.05\pi t))\mathbb{1}(t)$.

Second, consider the sampled-data LQR design for a regulated signal z such that

$$\|z(t)\|^2 = \|\dot{y}(t)\|^2 + \rho\|u(t)\|^2$$

for some $\rho > 0$.

4. Construct C_z and D_z for any regulated signal satisfying the equality above. Keep the state vector of the discretized plant unchanged. Under what conditions on ρ assumption $\mathcal{A}_3$ is satisfied?
5. Repeat the steps of Item 2 for the sampled-data design. Explain differences.
6. Repeat the steps of Item 3 for the sampled-data design.
7. Assume now that only the output $\bar{y}$ is measured. Under $h = 0.1\omega_d$, take $\rho = 1$, design the state-feedback gain via sampled-data LQR, and implement it in combination with the reduced-order dead-beat observer designed for the discretized plant, like in Homework 4. Simulate the analog closed-loop system for the square wave load disturbance $d(t) = \text{sign}(\sin(0.05\pi t))\mathbb{1}(t)$ and present two plots: that of the control signal and that of the analog output. The output plot should also depict the response of the uncontrolled plant to the same disturbance, by thin dashed lines.