



Sampled-Data Control Systems

Spring 2019/2020

Homework no. 4

(submission deadline: 9.06.2020, 11:59pm; please make an effort to be concise, clear, accurate, and rigorous)

Problem 1 (100%). Consider a pendulum on a cart having the transfer function

$$P(s) = \frac{2s^2}{(s/20 + 1)(s^2 + 0.02s + 25)}$$

This transfer function has one real pole at $s = -20$ and a pair of lightly-damped poles with the natural frequency $\omega_n = 5$ and the damping factor $\zeta = 1/500$, so that these poles are at $s = -\zeta\omega_n \pm j\sqrt{1 - \zeta^2}\omega_n$. The control goal is to dampen the pendulum against load (input) disturbances $d(t)$. In all simulations below consider the square wave of the unit amplitude and period 40 sec, i.e. $d(t) = \text{sign}(\sin(0.05\pi t))\mathbb{1}(t)$.

Consider first the digital redesign approach.

1. Let $h = \pi/20$ and choose the analog controller

$$C(s) = \frac{0.4917}{0.2705s + 1}$$

(designed for $P(s)(1 - e^{-sh})/(sh)$). Redesign this controller using the Tustin approximation and simulate the closed-loop analog response. Present two plots: that of the control signal and that of the analog output. Both plots should also depict, by dashed lines, the corresponding responses of the analog control system.

2. Repeat the steps of the previous item under $h = \pi/(\sqrt{1 - \zeta^2}\omega_n)$ and the P analog controller

$$C(s) = 0.1968.$$

Explain the results.

Now consider the discrete design.

3. Derive the state-space realization of the observer-based controller based on the *reduced-order observer* from Lect. 7, Slide 25.
4. Let $h = \pi/20$. Discretize the plant under the ZOH and design an observer-based stabilizing controller (as in the previous item) for it. In the design, choose the observer gain to obtain the dead-beat observer dynamics and choose the state-feedback gain to have the closed-loop poles in $\{e^{-20h}, \alpha, \alpha\}$ for some $|\alpha| < 1$. Tune α to end up with a *stable controller* $y \mapsto u$ and good damping in the continuous time. Present two plots: that of the control signal and that of the analog output. The output plot should also depict the responses of the output at the sampling instances, by dots ('.', 'MarkerSize', 20), and the response of the uncontrolled plant to the same disturbance, by dashed lines.
5. Repeat the steps of the previous item under $h = 3.3\pi/(\sqrt{1 - \zeta^2}\omega_n)$. Explain the results.

All continuous-time simulations should be carried out on the time interval $[0, 50]$ with the fixed-step solver under the step size $h/20$.