



Sampled-Data Control Systems

Spring 2019/2020

Homework no. 2

(submission deadline: 22.04.2020, 10:00am; please make an effort to be concise, clear, accurate, and rigorous)

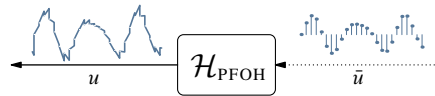
Problem 1 (16%). Consider the shift-invariant sampler $\mathcal{S}_a : y \mapsto \bar{y}$ acting as

$$\bar{y}[i] = \int_0^h e^{a(h-t)} c y(ih - t) dt$$

for some real a and $c \neq 0$.

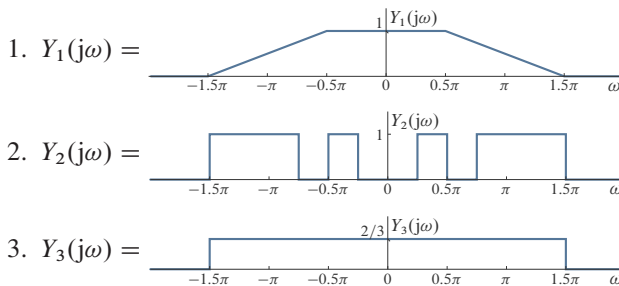
1. Is $\mathcal{S}_a$ causal? Static?
2. What is its sampling function $\psi(t)$? What is the transfer function of the corresponding F_ψ ? Plot the Bode diagram of the frequency response of F_ψ for $c = 1$ and $a \in \{-2, 0, 2\}$.

Problem 2 (24%). The *predictive first-order hold* $\mathcal{H}_{\text{PFOH}}$ is a shift-invariant D/A converter, which in the interval $[ih, (i+1)h)$ extends the straight line, connecting $\bar{u}[i-1]$ and $\bar{u}[i]$, see the figure below.



1. Derive a formula for the signal $u(t)$ produced by $\mathcal{H}_{\text{PFOH}}$.
2. Is $\mathcal{H}_{\text{PFOH}}$ causal? Static?
3. What is its hold function $\phi(t)$ (logic functions are not permitted)? Plot this function for some h of your choice. What is the transfer function of the corresponding F_ϕ ? Plot the Bode diagram of its frequency response.

Problem 3 (24%). Determine the spectra of the discrete signals, obtained by the ideal sampling with the sampling period $h = 2$ of analog signals having the following spectra:



Plot the resulting spectra and determine $\bar{Y}_i(1)$ for each one of them (indicate this point on each plot).

Problem 4 (36%). Consider the shift-invariant sampler $\mathcal{S}_{\text{sinc}}$ and hold $\mathcal{H}_{\text{sinc}}$, whose sampling and hold functions are

$$\psi(t) = h\phi(t) = \frac{\sin(\omega_N t)}{\omega_N t} =: \text{sinc}_h(t) = \text{sinc}_h(t), \quad \text{where } \omega_N := \frac{\pi}{h}.$$

1. Are these converters causal? Static?
2. What is the spectrum of the analog signal produces by $\mathcal{H}_{\text{sinc}}\mathcal{S}_{\text{sinc}}y$ for some y ? Plot it for

$$Y(j\omega) = \sqrt{\frac{2\pi^3}{(|\omega| + \pi)^3}} \quad (1)$$

and $h = 1$, together with the plot of $Y(j\omega)$.

3. Calculate $\|y - \mathcal{H}_{\text{sinc}}\mathcal{S}_{\text{sinc}}y\|_2$ for y as in (1) (an analytic expression is required). Plot the result as a function of h . Under what sampling period $\|y - \mathcal{H}_{\text{sinc}}\mathcal{S}_{\text{sinc}}y\|_2 \leq \|y\|_2/2$?