



Sampled-Data Control Systems

Spring 2019/2020

Homework no. 1

(submission deadline: 7.04.2020, 11:59pm; please make an effort to be concise, clear, accurate, and rigorous)

Problem 1 (12%). Suggest signals $f : \mathbb{Z} \rightarrow \mathbb{R}$ such that

1. $\|f\|_1 = \|f\|_2 = \|f\|_\infty = 1$,
2. $\|f\|_1 = \sqrt{3}$, $\|f\|_2 = \sqrt{2}$, and $\|f\|_\infty = 1$.

Problem 2 (8%). Suggest two functions $f_1 : \mathbb{Z} \rightarrow \mathbb{R}$ and $f_2 : \mathbb{Z} \rightarrow \mathbb{R}$ such that the RoCs of their z -transforms is the exterior of the unit disk, $\mathbb{C} \setminus \bar{\mathbb{D}}_1$, and the RoC of $f_1 + f_2$ is the whole complex plane, $\mathbb{C}$.

Problem 3 (14%). Suggest a definition for the equilibrium of a general nonlinear discrete system, described by

$$x[t+1] = f(x[t], u[t]).$$

Try to keep it consistent with its continuous-time counterpart. Find all equilibria of the (shifted) SIR model

$$\begin{cases} s[t+1] = s[t] - bs[t]i[t] \\ i[t+1] = i[t] + (bs[t] - a)i[t] + u[t] \\ r[t+1] = r[t] + ai[t] \end{cases}$$

under some $a > 0$ and $b > 0$.

Problem 4 (40%). Consider an LSI system G , whose transfer function $G(z) = 1 + 2z^{-5} + z^{-10}$.

1. What is the order of $G(z)$?
2. What are the poles of $G(z)$?
3. What are the zeros of $G(z)$?
4. Is $G(z)$ proper?
5. Is G causal?
6. What is the impulse response of G ? Plot it for $t = -13 : 13$
7. What is the frequency response of G ? Plot its Bode diagram.
8. Is G ℓ_2 -stable?
9. Is iG ℓ_∞ -stable?

Problem 5 (26%). The ideal low-pass filter F with the bandwidth $\theta_0 \in (0, \pi)$ is an LSI discrete system, whose frequency response

$$F(e^{j\theta}) = \begin{cases} 1 & \text{if } |\theta| \leq \theta_0 \\ 0 & \text{otherwise} \end{cases}.$$

1. Find its impulse response $f[t]$. Plot it for $\theta_0 = \pi/2$ and $t = -13 : 13$, use the `stem` command of Matlab.
2. Is F causal?
3. Is $f \in \ell_2(\mathbb{Z})$?
4. Let $\theta_0 = \pi/2$. Prove that this system is not ℓ_∞ -stable.