

תורת הבקרה (00350188)

סמסטר אביב תשפ"ה

פרויקט מס' 2

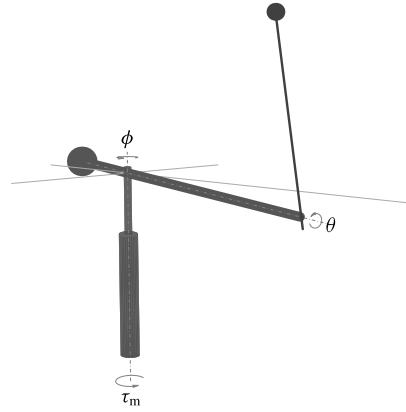


Figure 1: Schematic diagram of a Furuta pendulum in the upright position

1 Setup

A Furuta pendulum consists of an arm, which rotates in the horizontal plane, and a pendulum attached to it, which rotates in the vertical plane. The arm is mounted on a vertical shaft as shown in Fig. 1. The whole system is rotated by applying a torque τ_m to the shaft. The arm and pendulum angles, measured in radians, are denoted as ϕ and θ , respectively. We assume that the shaft angle equals that of the arm and that the zero pendulum angle corresponds to its upright position. The torque applied to the shaft is generated by a DC motor, so we assume that the rod includes the motor rotor and geared as its part.

2 Modeling

The linearized dynamics of the system around its equilibrium at $\phi = 0$, $\theta = 0$ (the upright position), and $\tau_m = 0$ are

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & J_b + mr^2 & 0 & -mrl \\ 0 & 0 & 1 & 0 \\ 0 & -mrl & 0 & J_p + ml^2 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -f & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & gml & -\zeta \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \tau_m, \quad (1)$$

where

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} := \begin{bmatrix} \phi \\ \dot{\phi} \\ \theta \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \text{arm angle} \\ \text{arm angular velocity} \\ \text{pendulum angle} \\ \text{pendulum angular velocity} \end{bmatrix}$$

and

- J_b and f are the total moment of inertia (around its pivot point) and viscous friction of the rotating base (rod + arm);
- r is the distance from the pendulum rotation plane to the arm pivot point;
- m is the pendulum mass;
- l is the distance from the pendulum pivot point to its centroid;
- J_p and ζ are the moment of inertia (around its centroid) and damping coefficient of the pendulum, respectively;
- g is the standard gravity.

Parameter	l	r	m	J_p	ζ	J_b	f	g	R	K_m	N_g	K_a
Value (in SI)	0.3226	0.3	0.04	0.0003	0.0005	0.01	0.01	9.81	2.08	0.0243	4.5	12

Table 1: Parameters of the system

The control torque τ_m is generated by a DC motor having the torque and back emf constants K_m and $K_b = K_m$, respectively, the resistance R , a negligible inductance, and a gearbox with the ratio N_g (so that the angular velocity of the motor shaft is $N_g \dot{\phi}$ and $\tau_m = N_g \tau_0$, where τ_0 is the torque generated by the rotor). The armature voltage of the motor is generated by an amplifier, modeled as a static gain K_a .

1. Derive, in a symbolic form, the state-space realization $(A_\tau, B_\tau, C_\tau, D_\tau)$ of the system $\tau_m \mapsto y := \begin{bmatrix} \phi \\ \theta \end{bmatrix}$, having x as its state vector (no need to resolve all relations there, it is enough to present the realization matrices as products of known matrices).
2. Derive, also in a symbolic form, the state-space realization (A, B, C, D) of the system $u \mapsto y$, where u is the amplifier input signal and whose state vector is still x .

Hereafter, use the parameters as in Table 1 in the derived model. Make sure that the resulting transfer function of the system $u \mapsto y := \begin{bmatrix} \phi \\ \theta \end{bmatrix}$ is

$$P(s) = D + C(sI - A)^{-1}B = \begin{bmatrix} 61.596(s - 5.27)(s + 5.382) \\ 53.43s^2 \end{bmatrix} \frac{1}{s(s + 6.451)(s - 5.908)(s + 1.144)}$$

(relevant Matlab commands are `P = ss(A, B, C, D)` and `zpk(P)`).

3. Check the controllability of (A, B) . If it is not controllable, then find uncontrollable modes and try to explain implications of their lack of controllability.
4. Check the observability of (C_ϕ, A) , where C_ϕ is such that $\phi = C_\phi x$. If it is not observable, then find unobservable modes and try to explain implications of their lack of observability.
5. Check the observability of (C_θ, A) , where C_θ is such that $\theta = C_\theta x$. If it is not observable, then find unobservable modes and try to explain implications of their lack of observability.
6. Is it possible to form a measured signal of the form $z = \mu' y = \mu_1 \phi + \mu_2 \theta$ for some constant μ_1 and μ_2 so that the mode of the system in the open right-half plane becomes unobservable? If it is, then write the transfer function $P_z(s)$ of the system $u \mapsto z$. Explain its qualitative difference from the components of $P(s)$.

3 State-feedback regulation

Assume that the whole state vector x is measurable and that the initial condition

$$x_0 = \begin{bmatrix} 0 \\ 0 \\ \pi/36 \\ 0 \end{bmatrix},$$

i.e. $\theta(0) = 5^\circ$ and all other state variables are in their equilibria.

7. Design a state-feedback control law placing all closed-loop modes to $s = -\alpha$, where $\alpha > 0$ is the largest number such that the resulting u , corresponding to the response of the closed-loop system to $x(0) = x_0$, satisfies $|u(t)| \leq 0.16$ for all $t \geq 0$ (by trial and error). Plot the response to the initial condition x_0 of ϕ , θ , u , and the Nichols plot of the frequency response of the loop $L(s) = -K(sI - A)^{-1}B$. Use the time grid $t = 0 : 0.01 : 5.2$. Find the stability margins of the resulted loop.
8. Design a state-feedback control law via formulating and solving an LQR problem. Specifically, consider the cost functions

$$\gamma_\phi = \int_0^\infty \phi^2(t)dt, \quad \gamma_\theta = \int_0^\infty \theta^2(t)dt, \quad \text{and} \quad \gamma_u = \int_0^\infty u^2(t)dt$$

and construct Q , S , and R for the LQR formulation minimizing

$$\mathcal{J} = \lambda \gamma_\phi + (1 - \lambda) \gamma_\theta + r \gamma_u$$

for some $\lambda \in (0, 1)$ and $r > 0$. Tune λ and r so that the resulting u , corresponding to the response of the closed-loop system to $x(0) = x_0$, satisfies $\max_t |u(t)| = 0.16$. Plot the response to the initial condition x_0 of ϕ , θ , u , and the Nichols plot of the frequency response of the loop $L(s) = -K(sI - A)^{-1}B$. Use the time grid $t = 0:0.01:5.2$. Find the stability margins of the resulted loop and compare them with the corresponding margins in Item 7. Explain the differences.

9. Write the transfer functions $C(s)$ of the state-feedback controllers in Items 7 and 8 considering ϕ and θ as their inputs. Interpret the components $C_\phi : \phi \mapsto u$ and $C_\theta : \theta \mapsto u$ in terms of classical PID controllers.

4 State-feedback command following

Still assume that the whole state vector x is measurable, but now set the initial condition to $x_0 = 0$. Our goal in this section is to design a reference trajectory and follow it in the 2DOF state-feedback configuration.

10. Design the minimum energy control signal $u = u_{\text{me}}$ that transfers the state of the system from $x(0) = 0$ to

$$x(t_1) = x_1 = \begin{bmatrix} \pi \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

for a given $t_1 > 0$. Also calculate the state trajectory $x = x_{\text{me}}$ generated by this control signal. Use the relations

$$\int_0^{t_1} e^{As} B B' e^{A's} ds e^{-A't_1} = \begin{bmatrix} I & 0 \end{bmatrix} \exp\left(\begin{bmatrix} A & B B' \\ 0 & -A' \end{bmatrix} t_1\right) \begin{bmatrix} 0 \\ I \end{bmatrix} \quad \text{and} \quad (e^{A't_1})^{-1} = e^{-A't_1}$$

to generate u_{me} and x_{me} in Matlab. Find, by trial and error, the shortest t_1 for which $|u_{\text{me}}(t)| \leq 0.16$ for all $t \geq 0$. Plot ϕ_{me} , θ_{me} (both are components of x_{me}), and u_{me} . Use the time grid $t = 0:0.01/N:t_1$ for some large enough $N \in \mathbb{N}$ (might require $N = 100$) to calculate the required trajectories, but then plot downsampled versions of the calculated signals, like `plot(t(1:N:end), u_me(1:N:end), 'LineWidth', 2)`, to reduce the size of plots.

11. For ϕ_{me} from the previous item, explain by common-sense physical arguments why the arm has to undershoot at the beginning of its motion and overshoot at the end.
12. With the state-feedback gain as in Item 8 and

$$x_{\text{ref}}(t) = \begin{cases} x_{\text{me}}(t) & \text{if } t \in [0, t_1] \\ x_1 & \text{otherwise} \end{cases} \quad \text{and} \quad u_{\text{req}}(t) = \begin{cases} u_{\text{me}}(t) & \text{if } t \in [0, t_1] \\ 0 & \text{otherwise} \end{cases}$$

for x_{me} and u_{me} designed in Item 10, design the 2DOF controller guaranteeing that $x = x_{\text{ref}}$. Calculate the resulted ϕ , θ , and u with the time grid $t = 0:0.01/N:t_1+0.3$ and plot them, together with ϕ_{ref} , θ_{ref} , and u_{req} , respectively, using downsampling like in Item 10.

Remarks

- All plots should be presented on separate figures, unless explicitly stated otherwise.
- Present all time-domain angle plots in *degrees*, even though the models assume them in radians.
- Consider using the code below (requires R2024b or newer) to produce Nichols charts with stability margins shown.

```
nichL = nicholsplot(L);
nichL.Title.String = "";
nichL.Grid = "on";
nichL.Responses.LineWidth = 2;
nichL.Characteristics.AllStabilityMargins.Visible = "on";
```
- Use `initial` to simulate initial condition responses and `lsim` to simulate responses to pre-calculated u_{req} and x_{ref} .
- It is advised to save plots using `exportgraphics(gca, 'name.pdf', 'ContentType', 'vector')` from within your Matlab code (requires R2020a or newer). This saves plots as vector PDF files without white margins.

כללי ההגשה

יש לשלוח את הפרוייקט לכתובת: ravina.l@campus.technion.ac.il. קובץ ההגשה, ארכיון מסוג ZIP או RAR ששמו הוא שם המגישים, יכיל קובץ PDF (יש להגיש דוח מודפס; סריקה של כתב יד לא תתקבל) המתאר את התשובות לשאלות השונות, שלבי הפתרון והסברים על פי הצורך. בנוסף, יכיל הארכיון את קבצי הקוד המקוריים, קבצי *.m, המבוארים באמצעות הערות ומחולקים בהתאם לשאלות. יש לרכז את כל החישובים בקובץ *.m יחיד. את הפרוצדורות לציור הגרפים ניתן להגיש בקבצי *.m * נוספים.