



Advanced Sampled-Data Control

Spring 2015/2016

Homework no. 5

(submission deadline: 15.08.2016, 10:00am)

Problem 1 (25%). Compute the FPR of the system $\mathcal{G}_e := 1 - \mathcal{H}_{\text{FOH}} \mathcal{S}_{\text{idl}}$, where $\mathcal{H}_{\text{FOH}}$ is the causal first-order hold, whose hold function is

$$\phi_{\text{FOH}}(t) = \left(1 + \frac{t}{h}\right) \mathbb{1}_{[0,h]}(t) + \left(1 - \frac{t}{h}\right) \mathbb{1}_{[h,2h]}(t).$$

Problem 2 (75%). Consider the following model of a pendulum mounted on a cart:

$$P(s) = -\frac{42s^2}{(s+18)(s^2+0.02s+23)}.$$

The control goal is to dampen the effect of input disturbance on the plant output (pendulum angle). To this end,

1. design an analog controller $K_0(s)$ minimizing the H^2 norm of the closed-loop system in the standard problem with the generalized plant

$$G(s) = \left[\begin{array}{cc|c} 0 & -P(s)W(s) & -P(s)W(s) \\ 0 & 0 & I \\ \hline I & -P(s)W(s) & -P(s)W(s) \end{array} \right],$$

where $W(s)$ is a *strictly proper* and stable weighting function (this follows the logic of the H^∞ loop shaping, but with the H^2 criterion and w/o the feedthrough term in G_{zw});

2. tune $W(s)$ so that the resulted closed-loop disturbance sensitivity transfer function

$$T_d(s) = \frac{P(s)}{1 + P(s)W(s)K_0(s)},$$

has its frequency response peak below 20 dB, i.e. such that $\|T_d\|_\infty \leq 10$;

3. use this design to parametrize all observer-based stabilizing controllers in the form $K(s) = W(s) \mathcal{F}_1(J_0(s), Q(s))$ with $K_0(s) = \mathcal{F}_1(J_0(s), 0)$;
4. digitally redesign this parametrization (its LFT part) and simulate the response of the resulting closed-loop system to a square-wave disturbance signal with the frequency 0.2π rad/sec and the magnitude 1 during 20 sec for sampling periods $h \in \{0.08, 0.16, 0.635, 1\}$ (it may be easier to simulate the sampled-data part of the controller directly in the form $\mathcal{F}_1(J_0, Q)$ with the Youla parameter Q reset to zero at ih , Simulink allows to add a “reset port” to any system);
5. calculate the FPR's of the closed-loop system in $\omega \in [1, 100]$ for each sampling period above and plot them together with $|T_d(j\omega)|^2$ for the analog design.