



Advanced Sampled-Data Control

Spring 2015/2016

Homework no. 4

(submission deadline: 16.06.2016, 10:00am)

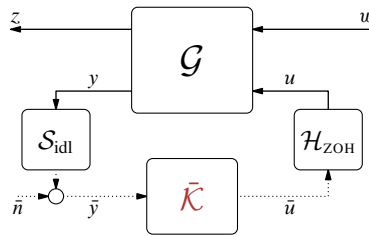


Figure 1: Sampled-data standard problem with discrete measurement noise

Problem 1 (33%). Consider the sampled-data problem in Fig. 1, where

$$G(s) = \left[\begin{array}{c|cc} A & B_w & B_u \\ \hline C_z & 0 & D_{zu} \\ C_y & 0 & 0 \end{array} \right]$$

and discrete measurement noise $\bar{n} = \Sigma^{1/2} \bar{w}_n$ for some $\Sigma \geq 0$ (it may account for round-off errors, for example). Solve the problem of finding a causal and stabilizing $\bar{K}$ that minimizes the H^2 -norm (define and interpret it) of the system $\mathcal{T}_{zw}$ from $\begin{bmatrix} w \\ \bar{w}_n \end{bmatrix}$ to z . To this end, rewrite the problem as a standard LTI H^2 problem in the lifted domain, use formulae from “Discrete-time output feedback” section of Lecture 9, and then peel the solution off. If some steps coincide with those of Lecture 9, don’t rederive the formulae.

Problem 2 (33%). Let

$$C(s) = \left[\begin{array}{c|c} A_c & B_c \\ \hline C_c & 0 \end{array} \right] \quad \text{and} \quad P(s) = \left[\begin{array}{c|c} A_p & B_p \\ \hline C_p & 0 \end{array} \right]$$

be minimal realizations of *stable* controller and plant, respectively, with $A_c \in \mathbb{R}^{n_c \times n_c}$ and $A_p \in \mathbb{R}^{n_p \times n_p}$. Consider the problem of approximating $\mathcal{C}$ by a sampled-data controller of the form $\mathcal{H}_{\text{ZOH}} \bar{\mathcal{C}} \mathcal{S}_{\text{idl}}$ via minimizing

$$\|(\mathcal{C} - \mathcal{H}_{\text{ZOH}} \bar{\mathcal{C}} \mathcal{S}_{\text{idl}}) \mathcal{P}\|_2$$

by a stable discrete $\bar{\mathcal{C}}$. To this end, formulate the problem as a standard sampled-data H^2 problem of Type I.

1. Present the corresponding continuous-time generalized plant $\mathcal{G}$ and its state-space realization (make sure its state dimension doesn’t exceed $n_c + n_p$).
2. Present the lifted generalized plant and its state-space realization.
3. Which of the four solvability conditions hold in this problem regardless parameters of $P(s)$ and $C(s)$?
4. What is the order of the resulted $\bar{\mathcal{C}}(z)$?
5. Solve the problem with

$$C(s) = -\frac{2.1581(s + 0.2895)}{s^2 + 1.91s + 1.514} \quad \text{and} \quad P(s) = -\frac{42s^2}{(s + 18)(s^2 + 0.02s + 23)}$$

and find the maximal sampling period h for which the closed-loop system remains stable. What is the order of the resulted $\bar{\mathcal{C}}(z)$ (provided all pole-zero cancellations are carried out)?

Problem 3 (34pt). Consider the approximation problem of Problem 2 under the same assumptions. Now, solve it as a model-matching problem in the lifted domain, i.e. without using the solution to the standard problem, explicitly performing the required spectral factorizations and partial fraction expansions (and don't expect to have $\tilde{Q}_{\text{opt}}(z) = 0$). Write down its solvability conditions (in terms of finite-dimensional matrices), the optimal $\tilde{C}(z)$, and the optimal attainable performance. How many Riccati equations have to be solved? What is the order of the optimal approximation? Apply the formulae to the case in item 5 of Problem 2.