



Advanced Sampled-Data Control

Spring 2015/2016

Homework no. 3

(submission deadline: 24.05.2016, 10:00am)

Problem 1 (25%). Proof (lifted Parseval) that for all $f_1, f_2 \in L_2(\mathbb{R})$

$$\sum_{i \in \mathbb{Z}} \langle \check{f}_1[i], \check{f}_2[i] \rangle_{\mathbb{L}} =: \langle \check{f}_1, \check{f}_2 \rangle_2 = \langle \check{F}_1, \check{F}_2 \rangle_2 := \frac{1}{2\pi} \int_{-\pi}^{\pi} \langle \check{F}_1(e^{j\theta}), \check{F}_2(e^{j\theta}) \rangle_{\mathbb{L}},$$

where $\check{f}_i = \mathcal{L}_h f_i$, $\check{F}(e^{j\theta})$ is the lifted Fourier transform of $\check{f}_i$, and $\mathbb{L} := L_2[0, h)$.

Problem 2 (25pt). Let $\mathcal{G}$ be an LTI continuous-time causal system with the transfer function $G(s) = C(sI - A)^{-1}$. Find the kernel $\check{G}(e^{j\theta}; \tau, \sigma)$ of the lifted frequency response of $\check{\mathcal{G}} = \mathcal{L}_h \mathcal{G} \mathcal{L}_h^{-1}$. Show that it is a rational function of $e^{j\theta}$ for any given $\tau, \sigma \in [0, h)$.

Problem 3 (25%). Given an LTI continuous-time system $\mathcal{K} \in L^\infty \cap L_2$, consider its approximation by a sample-and-hold circuit of the form $\mathcal{H} \tilde{\mathcal{K}} \mathcal{S}$ for given $\mathcal{H} \in L^\infty$ and $\mathcal{S} \in L^\infty$. Find a $\tilde{\mathcal{K}} \in L^\infty \cap L_2$ that minimizes $\|\mathcal{K} - \mathcal{H} \tilde{\mathcal{K}} \mathcal{S}\|_2$, its frequency response, and the optimal attainable norm.

Problem 4 (25pt). Solve the approximation problem of the previous question in the case when $K(j\omega) = \frac{1}{j\omega(j\omega+1)}$, $\mathcal{H} = \mathcal{H}_{\text{ZOH}}$ with the hold function $\phi(t) = \mathbb{1}_{[0,h]}(t)$, and the sampler $\mathcal{S}$ has the sampling function $\psi(t) = e^{-t} \mathbb{1}(t)$. Present plots of the impulse response $\tilde{k}_{\text{opt}}[i]$ of the resulting $\tilde{\mathcal{K}}_{\text{opt}}$ and the optimal attainable performance as a function of the sampling period h . (Ignore the fact that $\mathcal{K} \notin L^\infty \cap L_2$ and that the resulting $\tilde{\mathcal{K}}$ won't be stable either, just apply the formulae of the previous problem.)