



Advanced Sampled-Data Control

Spring 2015/2016

Homework no. 2

(submission deadline: 5.05.2016, 10:00am)

Problem 1 (25%). Given a causal LTI continuous-time system $\mathcal{G}$ having the impulse response $g(t)$, construct $\bar{\mathcal{G}} = \mathcal{S}_{\text{idl}}\mathcal{G}\mathcal{H}_{\text{FOH}}$. When is it causal? Then, assuming $G(s) = D + C(sI - A)^{-1}B$ with $B \in \mathbb{R}^{n \times m}$, construct a state-space realization of $\bar{G}(z)$ and express its parameters using *one* $(n + 2m) \times (n + 2m)$ -dimensional matrix exponential.

Remark: When we write $\int_{\mathbb{R}}(\cdot)dt = \sum_i \int_{ih}^{(i+1)h} f(t)dt$, we mean that each term is actually $\lim_{\epsilon \downarrow 0} \int_{ih}^{(i+1)h-\epsilon} f(t)dt$.

Problem 2 (25%). Given a causal LTI continuous-time system $\mathcal{G}$ having the transfer function $G(s) = C(sI - A)^{-1}B$ (minimal realization with full column rank B and full row rank C). Construct $\bar{\mathcal{G}} = \mathcal{S}_{\text{idl}}\mathcal{G}\mathcal{H}_k$ and its state-space realization, assuming that $\mathcal{H}_k$ has the hold function $\phi_k(t) = B'e^{A'(h-t)}W_c^{-1}(h)\mathbb{1}_{[0,h]}(t)$, where

$$W_c(t) := \int_0^t e^{A\tau}BB'e^{A'\tau}d\tau > 0.$$

Find a system $\bar{\mathcal{G}}^\#$, such that $\bar{\mathcal{G}}^\# \in H^\infty$ and $\bar{G}(z)\bar{G}^\#(z) = z^{-1}I$.

Problem 3 (25pt). Consider analog plant $\mathcal{P}$ and controller $\mathcal{K}$ having the transfer functions

$$P(s) = -\frac{42s^2}{(s+18)(s^2+0.02s+23)} \quad \text{and} \quad K(s) = -\frac{2.1581(s+0.2895)}{s^2+1.91s+1.514}.$$

Assuming that A/D and D/A converters are the ideal sampler and the zero-order hold, respectively, redesign this controller using both the Tustin's and step-invariant (where $\bar{\mathcal{K}} = \mathcal{S}_{\text{idl}}\mathcal{K}\mathcal{H}_{\text{ZOH}}$) approximations. First, do it with $h = 0.02$ and present the *analog* closed-loop responses to the analog input disturbance $d(t) = \text{sign}(\sin(\frac{\pi}{5}t))\mathbb{1}(t)$ (square wave) under the analog and each of the redesigned sampled-data controllers. For each approximation method, find the maximal h for which the closed-loop system is stable. Present simulations for these redesigns as well.

Problem 4 (25pt). Consider the analog plant and controller in the previous problem and let $T_d(s) = \frac{P(s)}{1+P(s)K(s)}$ be the corresponding disturbance sensitivity transfer function. For each $h \in \{0.02, 0.08, 0.16, 0.635\}$, construct the step-invariant discretization $\bar{P}(z)$ of $P(s)$ and design for it a 2-order discrete controller, which places the closed loop poles to the poles of the step-invariant discretization $\bar{T}_d(z)$ of $T_d(s)$ for a given h (use the method based on the Sylvester matrix, see <http://leo.technion.ac.il/Courses/CT/Lectures/lect04handout.pdf>). Simulate *analog* closed-loop response for each of these designs under the same disturbance as in the previous problem.