



Advanced Sampled-Data Control

Spring 2015/2016

Homework no. 1

(submission deadline: 14.04.2016, 10:00am)

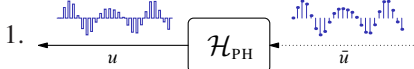
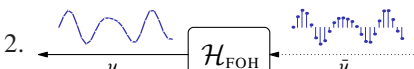
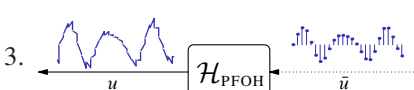
Problem 1 (20%). Prove that every linear D/A converter can be described as

$$u(t) = \sum_{i \in \mathbb{Z}} \phi(t, t_i) \bar{u}[i]$$

for some $\phi(t, t_i)$ (the hold function). Interpret the hold function.

Problem 2 (25pt). Prove that the first-order hold associated with the sampling pattern $\{t_i\}_{i \in \mathbb{Z}}$ is stable whenever sampling intervals are uniformly bounded, i.e. that $\mathcal{H}_{\text{FOH}} : \ell^2(\mathbb{Z}) \rightarrow L_2(\mathbb{R})$, and that $\|u\|_2^2 \leq \sup_i h_i \|\bar{u}\|_2^2$ for every $\bar{u} \in \ell^2(\mathbb{Z})$.

Problem 3 (30%). Find the induced $\ell^2(\mathbb{Z}) \rightarrow L_2(\mathbb{R})$ gains of the following hold functions (in all cases assume uniform sampling with the sampling period $h > 0$ and stability):

1.  : produces pulses of height $\bar{u}[i]$ and width αh for some $\alpha \in (0, 1)$
2.  : the linear interpolation between $\bar{u}[i]$ and $\bar{u}[i + 1]$
3.  : extends to $[t_i, t_{i+1}]$ the linear interpolation between $\bar{u}[i - 1]$ and $\bar{u}[i]$

Problem 4 (25pt). Assuming uniform sampling with the sampling period $h > 0$, calculate the $L_2(\mathbb{R}) \rightarrow L_2(\mathbb{R})$ induced norm of the sampled-data controller $\mathcal{H}_{\text{ZOH}} \tilde{\mathcal{K}} \mathcal{S}_{\text{avg}}$, where $\mathcal{H}_{\text{ZOH}}$ is the zero-order hold, $\tilde{\mathcal{K}}$ is an LTI system with the transfer function $\tilde{K}(z) = 1 + z^{-1}$, and $\mathcal{S}_{\text{avg}}$ is the averaging sampler with $\Delta = h$.