הטכניון – מכון טכנולוגי לישראל, הפקולטה להנדסת מכונות

TECHNION—Israel Institute of Technology, Faculty of Mechanical Engineering

Introduction to Control (034040)

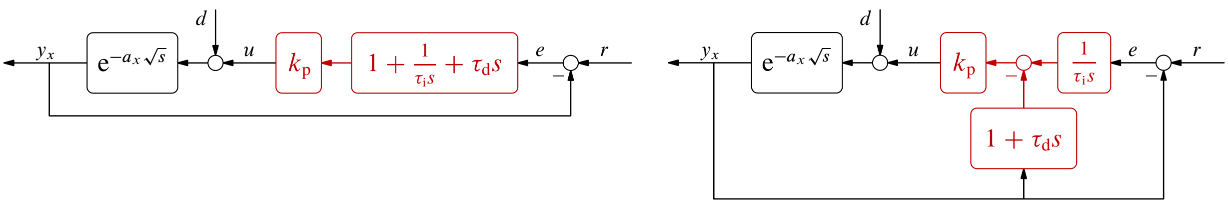
TUTORIAL 12

Question 1. Consider the one-dimensional heat propagation in a semi-infinite rod studied in Tutorial 10, where the control signal is the temperature u at one end and that the output is the temperature y_x at a point along the rod at distance $x > 0\,[\mathrm{m}]$ from the end. The transfer function of this system is

$$P_x(s) = \mathrm{e}^{-a_x\sqrt{s}}, \quad \text{where } a_x := \frac{x}{\sqrt{\alpha}} > 0 \tag{1}$$

where $\alpha > 0\,[\mathrm{m}^2/\mathrm{s}]$ is the rod thermal diffusivity. We assume that at $t < 0$ the system is in its equilibrium with $u(t) = y_x(t) = 20°\mathrm{C}$.

1. What K_u and T_u are produced by the Ziegler-Nichols closed-loop experiment ?
2. Design a P controller by the Ziegler-Nichols table. Analyze the resulting stability margins. Then, assuming that $\alpha = 1.27\cdot 10^{-4}\,[\mathrm{m}^2/\mathrm{s}]$ (corresponds to a golden rod) and $x = 0.1\,[\mathrm{m}]$, simulate the system response to the step reference change from 20° to 30° at $t = 0$, i.e. $r = 20 + 10\cdot\mathbb{1}$ and the step disturbance $d = \mathbb{S}_{-120}\mathbb{1}$, which may reflect a failure in the actuator (heater) at $t = 2\,[\mathrm{min}]$.
3. Under the same conditions, design a PI controller, analyze it, and simulate its response under the unity-feedback implementation scheme presented in Fig. 1(a). Compare the response with that under the implementation scheme presented in Fig. 1(b).
4. Under the same conditions, design a PID controller, analyze it, and simulate its response under the unity-feedback implementation scheme presented in Fig. 1(a). Compare the response with that under the implementation scheme presented in Fig. 1(b).



(a) Unity-feedback implementation

(b) Alternative feedback implementation (PI or PID)

Fig. 1: Closed-loop temperature control in a semi-infinite rod

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Question 2. Consider a system consisting of a plant and a feedback controller given by their transfer functions

$$P(s) = \frac{750}{s(s+5)(s+10)} \quad \text{and} \quad C(s) = 2.9303\left(\frac{1.8455s + 11.5}{s + 21.2228}\right)^2 \frac{10s + 11.5}{10s + 0.0989},$$

respectively. When implemented in the unity-feedback architecture, this system was studied in Tutorial 11 (the second design in Question 1) and has a closed-loop bandwidth of about 20 [rad/sec], a phase margin of 35°, and steady-state error to a step disturbance of 1% of the disturbance magnitude. The transient command response for this system was not quite exciting, with an overshoot of 42% and a settling time of 1.84 [sec] under settling level 1% (we could have expected shorter transients for this bandwidth).

To improve the command response, consider the use of the 2DOF configuration in Fig. 2, where the reference model T_{ref} is stable and such that $C_{\mathrm{ol}} = P^{-1}T_{\mathrm{ref}}$ is stable as well.

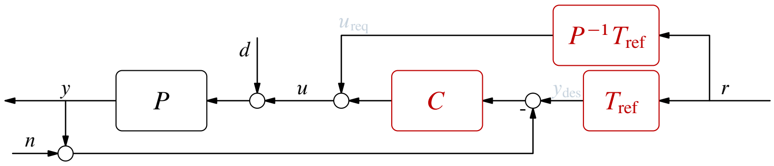

Fig. 2: 2DOF control configuration for the system in Question 2

1. Derive the relation between the exogenous signals r, d, and n and the signals of interest y and u.

2. Choose a reference model of the form

$$T_{\mathrm{ref}}(s) = \frac{1}{(\tau s + 1)^n}$$

having the minimum possible degree n and such that the high-frequency gain of the system $r \mapsto u$ is the same as the high-frequency gain of the control sensitivity, $T_{\mathrm{c}}(\infty)$. Simulate the system with $r = \mathbb{1}$ and $d = -0.2\mathbb{S}_{-1.84}\mathbb{1}$. Compare the results with those obtained for the unity-feedback (1DOF) implementation of the control system.

3. Suggest an admissible reference model T_{ref}, which has $T_{\mathrm{ref}}(0) = 1$, satisfies the requirements of the previous item, and renders the block $P^{-1}T_{\mathrm{ref}}$ simplest (i.e. results in its lowest possible degree), which simplifies its implementation. Simulate the resulting closed-loop system.