Introduction to Control (034040)

tutorial 10

Question 1. Fig.1 presents the Bode plot of a system having the transfer function

$$P(s) = \frac{k\mathrm{e}^{-\tau s}}{s+1}$$

for some $k > 0$ and $\tau > 0$. Determine the static gain k and the delay $\tau > 0$.

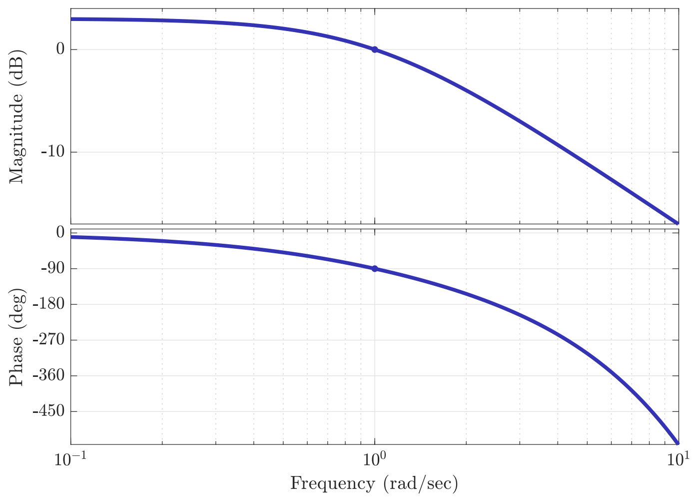

Fig. 1: Bode plot of P in Question 1

Question 2. Consider the problem of one-dimensional heat propagation in a semi-infinite rod. Assume that the input is the temperature u at one end and that the output is the temperature y_x at a point along the rod at distance $x > 0$ from the end. Let $\theta(x,t)$ denote the temperature at the position x and time t (so $y_x(t) = \theta(x,t)$). Heat propagation in this system is described by the parabolic partial differential equation

$$\frac{\partial\theta(x,t)}{\partial t} = \alpha\frac{\partial^2\theta(x,t)}{\partial x^2}, \qquad \theta(0,t) = u(t) \tag{1}$$

where $\alpha > 0\,[\mathrm{m}^2/\mathrm{s}]$ is a parameter, known as *thermal diffusivity*, which depends on properties of the rod.

1. Assuming that θ is bounded for every bounded u (which agrees with the physics), derive the transfer function $u \mapsto y_x$ and the corresponding impulse responses. Check the BIBO stability of the system.
2. Construct the Bode and polar plots of the system.
3. Let the system be controlled by a unity-feedback P controller $C(s) = k_\mathrm{p}$.
   (a) Under what $k_\mathrm{p} > 0$ the closed-loop system is stable?
   (b) What is the upper bound on the attainable crossover frequencies under stabilizing k_p's?
   (c) What is the lower bound on the attainable steady-state error?
   (d) What are the stability margins for this loop as functions of x, α, and k_p?
4. Let the system be controlled by a unity-feedback I controller $C(s) = \frac{k_\mathrm{i}}{s}$. Under what $k_\mathrm{i} > 0$ the closed-loop system is stable?