הטכניון – מכון טכנולוגי לישראל, הפקולטה להנדסת מכונות
TECHNION—Israel Institute of Technology, Faculty of Mechanical Engineering

Introduction to Control (034040)

tutorial 8

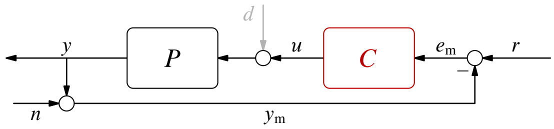

Fig. 1: Unity-feedback control system

Question 1. Consider the unity-feedback system in Fig. 1. Are the requirements below contradictory?

1. $\begin{cases} |E_r(\mathrm{j}\omega)| < 0.1|R(\mathrm{j}\omega)| & \text{for all } \omega < 10 \\ |E_n(\mathrm{j}\omega)| < 0.1|N(\mathrm{j}\omega)| & \text{for all } \omega > 1 \end{cases}$

2. $\begin{cases} |E_r(\mathrm{j}\omega)| < 0.1|R(\mathrm{j}\omega)| & \text{for all } \omega < 1 \\ |E_n(\mathrm{j}\omega)| < 0.1|N(\mathrm{j}\omega)| & \text{for all } \omega > 10 \end{cases}$

where e_r is the effect of the reference signal r on the tracking error $e := r - y$, e_n is the effect of the measurement noise n on it, and $X(\mathrm{j}\omega)$ stands for the value of the spectrum of a signal x at a frequancy ω.

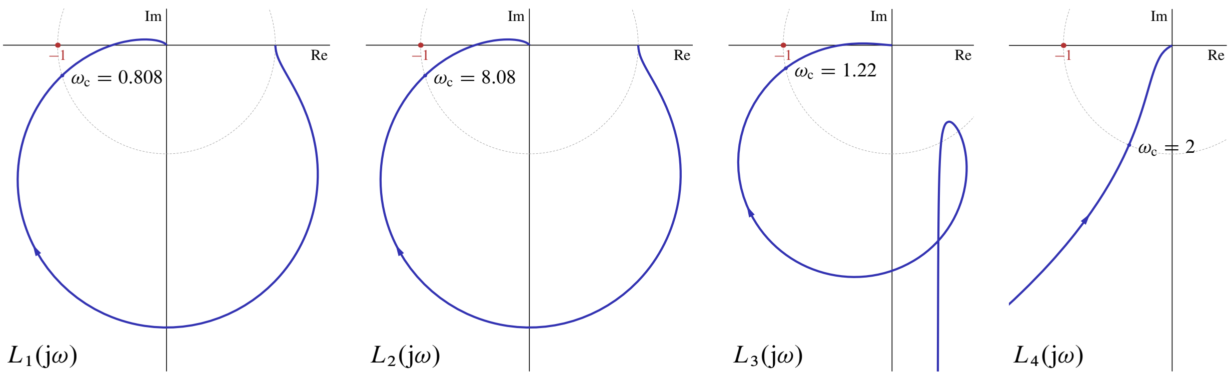

Fig. 2: Polar plots of $L_i(\mathrm{j}\omega)$ for Question 2

Question 2. Fig. 2 depicts the polar plots of four loop frequency responses $L_i(\mathrm{j}\omega) = P_i(\mathrm{j}\omega)C_i(\mathrm{j}\omega)$ for $i = 1, \ldots, 4$. These systems are controlled in the standard unity-feedback configuration in Fig. 1.

1. Fig. 3 depicts the Bode plots of the closed-loop complementary sensitivity functions T_k, $k = 1, \ldots, 4$. Relate between k and i.

2. Fig. 4 depicts the step responses of the closed-loop sensitivity functions $S_l(s)$, $l = 1, \ldots, 4$. Relate between l and i.

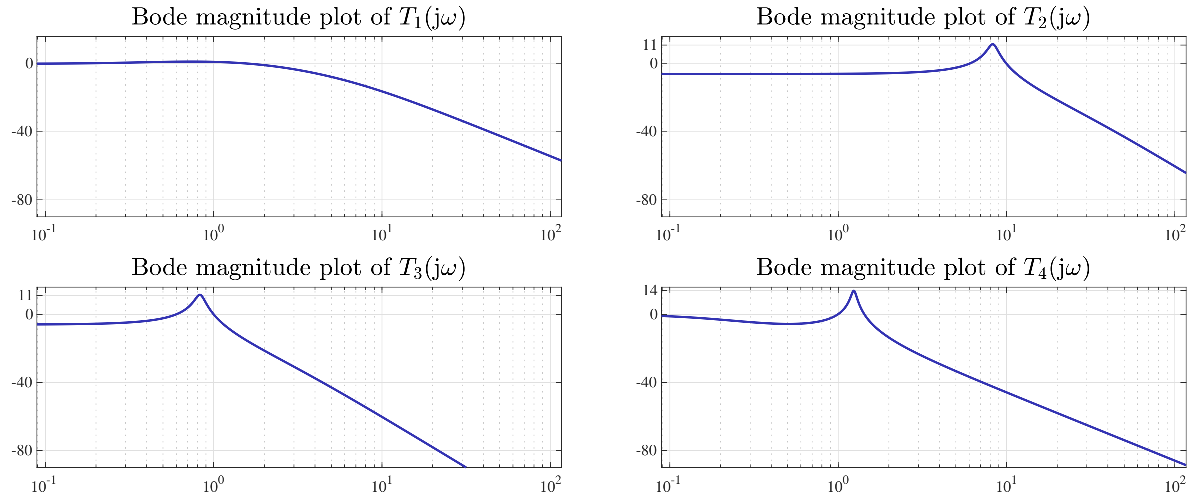

Fig. 3: Bode magnitude plots of $T_k(\mathrm{j}\omega)$ for Question 2

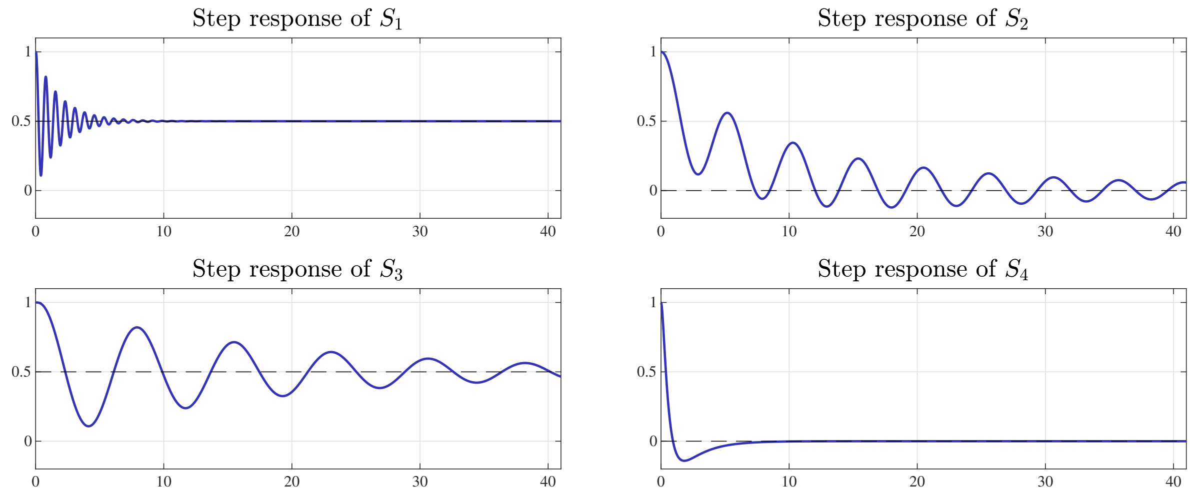

Fig. 4: Step responses of S_l for Question 2

Question 3. Consider yet again the problem of cruise control for a vehicle, like that depicted in Fig. 5. The problem here is to maintain the vehicle velocity v at prespecified levels by changing the driving force f generated by the engine. As we already know (Tutorials 3 and 5), the linearized (around some given velocity v_{eq}) model of this system is

$$y = \frac{1}{ms + \alpha v_{\mathrm{eq}}} u$$

where m is the mass of the vehicle, α is a constant depending on the density of air, the frontal area of

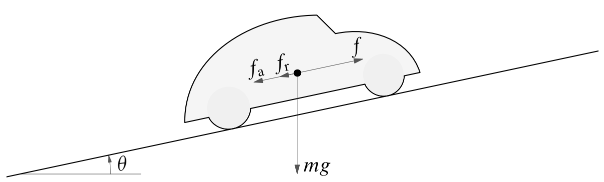

Fig. 5: System for Question 3

the car, and a shape-dependent aerodynamic drag coefficient, and the deviation variables $y := v - v_{\mathrm{eq}}$ and $u := f - 0.5\alpha v_{\mathrm{eq}}^2 - mg(\sin\theta + C_r\cos\theta)$, where C_{r} is the (dimensionless) rolling resistance coefficient and θ is the actual road slope. In all numerical calculations of what follows we assume the following numerical values:

α	C_{r}	g	m_0	θ_0	v_{eq}
1 [kg/m]	0.01	9.8 [m/sec^2]	1000 [kg]	$12° \approx 0.20944$ [rad]	80 [km/h] $\approx$ 22.2222 [m/sec]

Assume that the system is controlled by in the unity-feedback scheme, like that in Fig. 1 by a proportional controller $C(s) = k_{\mathrm{p}} > 0$

1. Normalize the control signal and the output such that the plant has the unit static gain.
2. Find the bandwidth of the normalized plant and of the closed-loop complementary sensitivity function T (as function of k_{p}).
3. Find the control effort at $t = 0$. Express the closed-loop bandwidth $\omega_{\mathrm{b},T}$ as a function of $\bar{u}(t \to 0)$ and the bandwidth of the normalized plant.
4. Draw the Bode plots of $\bar{T}(\mathrm{j}\omega)$, $\bar{P}(\mathrm{j}\omega)$, and $\bar{T}_{\mathrm{c}}(\mathrm{j}\omega)$ for $k_{\mathrm{p}} \in \{500, 2000, 6000\}$. Explain the results of section 2 based on the resulting plots.