Introduction to Control (034040)

tutorial 7

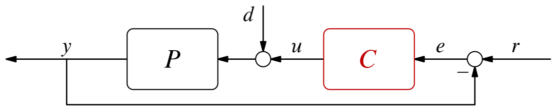

Fig. 1: Unity feedback closed-loop system

Question 1 (self-study)**.** A process with the transfer function $P(s) = 1/((s+1)(s+2))$ is controlled in closed loop with unity feedback as in Fig. 1.

1. Sketch the root locus of the system under a proportional (P) controller, i.e. $C(s) = k_p > 0$.
2. Find regions of the P controller gain k_p for which the closed-loop response $r \mapsto y$ satisfies the following specifications:
   (a) the overshoot OS $\le 10\%$,
   (b) the natural frequency $\omega_n \in [5/3, 8/3]$ [rad/sec].
3. Calculate the steady-state errors to step r and d as functions of the P controller gain k_p. What are the smallest errors under admissible controllers from the previous item?
4. Consider now a proportional-integral (PI) controller of the form $C(s) = k_p(1 + k_i/s)$ for some $k_p > 0$ and $k_i > 0$. Calculate the steady-state errors to step r and d as functions of the PI controller parameters k_p and k_i.

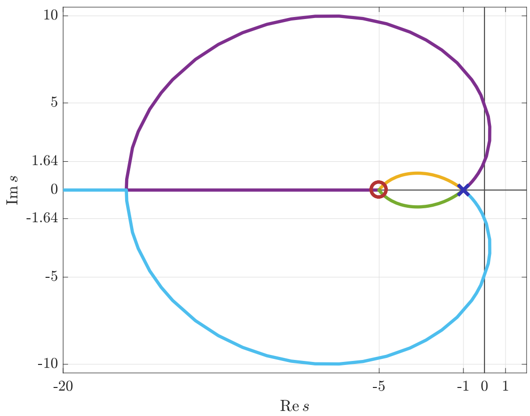

Fig. 2: Root locus system 1

Question 2. Fig. 2 depicts the root-locus of a plant P controlled in the unity-feedback configuration in Fig. 1 with a proportional controller, $C(s) = k_p$.

1. Can $P(s)$ be determined unambiguously?

2. Assuming that the leading coefficient of the numerator of $P(s)$ is positive (the denominator is monic), determine the sign of k_p in Fig. 2 and sketch the root-locus plot for the complementary region.

3. Fig. 3 presents the closed-loop step responses for $k_p = 0.093$, $k_p = -0.008$, and $k_p = 1.885$. Match the time-domain response to each of these controller gains. What are the steady-state frequencies of the oscillating responses?

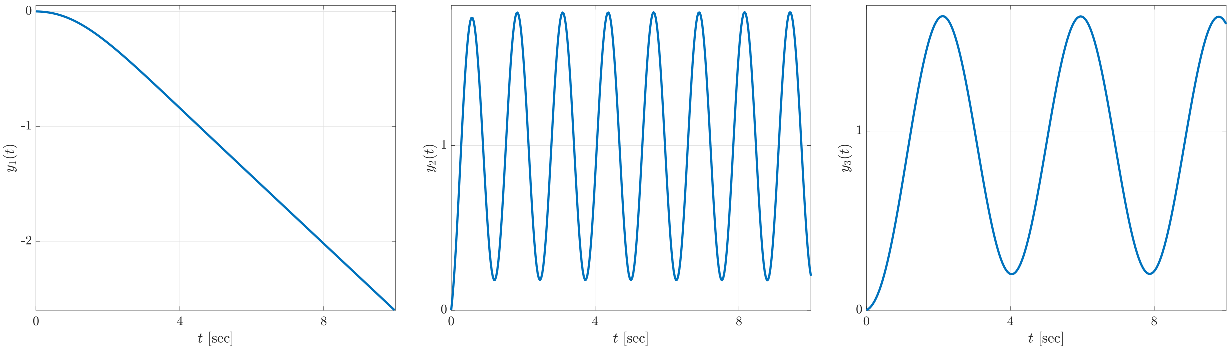

Fig. 3: Step responses of the closed-loop system for different k_p

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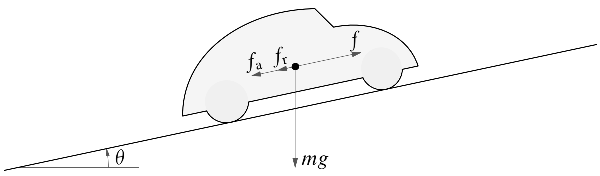


Fig. 4: System for Question 3

Question 3. Consider again the problem of cruise control for a vehicle, like that depicted in Fig. 4. The problem here is to maintain the vehicle velocity v at prespecified levels by changing the driving force f generated by the engine. As we already know (Tutorials 3 and 5), the linearized (around some given velocity v_{eq}) model of this system is

$$y = \frac{1}{ms + \alpha v_{eq}}(u + d),$$

where m is the actual mass of the vehicle, α is a constant depending on the density of air, the frontal area of the car, and a shape-dependent aerodynamic drag coefficient, and the deviation variables $y := v - v_{eq}$ and $u := f - 0.5\alpha v_{eq}^2 - m_0 g(\sin\theta_0 + C_r \cos\theta_0)$, where C_r is the (dimensionless) rolling resistance coefficient, m_0 is the *assumed* mass of the vehicle and θ_0 is the *assumed* slope angle of the road. The disturbance signal, which accounts for inaccuracies in the assumed mass and angle, satisfies

$$d(t) = d_0 := m_0 g(\sin\theta_0 + C_r \cos\theta_0) - mg(\sin\theta + C_r \cos\theta),$$

where θ is the actual road slope. In all numerical calculations of what follows we assume the following numerical values:

α	C_r	g	m_0	θ_0	v_{eq}
1 [kg/m]	0.01	9.8 [m/sec^2]	1000 [kg]	$12° \approx 0.20944$ [rad]	80 [km/h] ≈ 22.2222 [m/sec]

Assume that the system is controlled by in the unity-feedback scheme, like that in Fig. 1. We saw in Tutorial 6 that a proportional controller is not capable to attain a required new velocity v_{new}. Consider now another controller, PI:

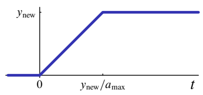
$$C(s) = k_{\text{p}}\Bigl(1 + \frac{k_{\text{i}}}{s}\Bigr) \tag{1}$$

for some parameters $k_{\text{p}} > 0$ and $k_{\text{i}} > 0$.

1. Sketch the root-locus plot for the system with respect to the proportional gain k_{p}. How the choice of k_{i} affects it? Under what k_{p} and k_{i} the closed-loop system is stable?

2. What is the steady-state velocity in this case under

$$r_v(t) = \begin{cases} 0 & \text{if } t \le 0 \\ a_{\max} t & \text{if } 0 \le t \le y_{\text{new}}/a_{\max} \\ y_{\text{new}} & \text{if } t \ge y_{\text{new}}/a_{\max} \end{cases} = \tag{2}$$



   for the peak acceleration $a_{\max} = 0.5\,[\text{m/s}^2]$ and $y_{\text{new}} = 10\,[\text{km/h}] = 25/9 \approx 2.78\,[\text{m/sec}]$ under $m = m_0$ and $\theta = \theta_0$?

3. How the choices of k_{p} and k_{i} affect the steady-state error in general, under $m \ne m_0$ and $\theta \ne \theta_0$? Simulate the response of the linearized closed-loop system to r as in (2) under the nominal car mass and a step change of the road slope at $t = 13\,[\text{sec}]$ to $\theta = 13° \approx 0.2269\,[\text{rad}]$ under $k_{\text{p}} \in \{500, 1000, 5000\}$ and $k_{\text{i}} \in \{0.0222, 0.1331\}$.

4. Analyze the nonlinear system with the unity feedback closed-loop PI controller as in (1) (the nonlinear plant dynamics are $m\dot{v} = f - 0.5\alpha v^2 - mg(\sin\theta + C_{\text{r}}\cos\theta)$). What is its steady-state response to the reference signal in (2)?