הטכניון – מכון טכנולוגי לישראל, הפקולטה להנדסת מכונות

TECHNION—Israel Institute of Technology, Faculty of Mechanical Engineering

Introduction to Control (034040)

tutorial 6

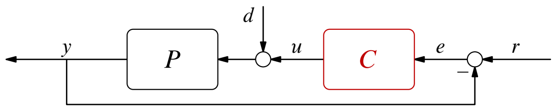

Fig. 1: Unity feedback closed-loop system

Question 1. Consider the unity feedback system in Fig. 1. Sketch (qualitatively) the root-loci of the following systems with respect to the gain k. Is it possible to stabilize the system with $k > 0$?

1. $P(s) = \dfrac{s+1}{(s-1)(s-2)(s+5)}$ and $C(s) = k$
2. $P(s) = \dfrac{(s-1)(s+1)}{(s-3)(s-5)}$ and $C(s) = k$
3. $P(s) = \dfrac{1}{(s-1)^3}$ and $C(s) = k\,\dfrac{s-3}{s-7}$
4. $P(s) = \dfrac{1}{s^2}$ and $C(s) = \dfrac{s+k}{s+2}$

Question 2. Consider the system $P(s) = (s-1)/(s-2)$ controlled in closed loop with unity feedback.

1. Can the system be stabilized by a proportional controller of the form $C(s) = k_\mathrm{p}$ for some $k_\mathrm{p} > 0$?
2. Can the system be stabilized by a *stable* controller, i.e. $C(s) = k_\mathrm{p} N_C(s)/D_C(s)$ for some Hurwitz $D_C(s)$?
3. Discuss the requirements for a controller of the form $C(s) = k_\mathrm{p}\tilde{C}(s)$ for stabilizing the system. Considering the controller $C(s) = k_\mathrm{p}/(s-a)$, for some $k_\mathrm{p} > 0$, find the requirements on a
   (a) by analyzing roots of the closed-loop characteristic polynomial,
   (b) by using root-locus principles.
4. Find the range of k_p for which the closed-loop system is stable for the controller in the previous item.

Question 3 (self-study)**.** Consider an inverted pendulum which consists of a point mass m on a mass-less rod of length l installed on a cart of mass M. An external force $u(t)$ is acting on the cart. The controlled output is assumed to be the acceleration of the cart, $y(t) = \ddot{z}(t)$. We know from Tutorial 6 that the transfer function of this system, linearized around the pendulum "up" position is (here g is the standard gravity)

$$P(s) = \frac{ls^2 - g}{Mls^2 - (M+m)g} = \frac{0.1(s-3.13)(s+3.13)}{(s-3.834)(s+3.834)},\ P(s) = \frac{ls^2 - g}{Mls^2 - (M+m)g} = \frac{1}{M}\,\frac{s^2 - \gamma^2}{s^2 - \mu^2\gamma^2},$$

where

$$\gamma := \sqrt{\frac{g}{l}} > 0 \quad \text{and} \quad \mu := \sqrt{1 + \frac{m}{M}} > 1.$$

Consider the control of this system by the unity-feedback system like that in Fig. 1.

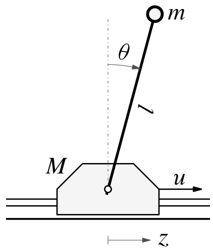

Fig. 2: Inverted pendulum on a cart for Question 3

1. Is it possible to stabilize this system by a proportional controller, i.e. by $C(s) = k_{\mathrm{p}}$ for some $k_{\mathrm{p}} > 0$? Explain via root-locus arguments.

2. Is it possible to stabilize this system by a *stable* controller having the positive high-frequency gain, i.e. such that
$$C(s) = \frac{k_{\mathrm{p}} N_C(s)}{D_C(s)} = \frac{k_{\mathrm{p}}(s^m + +b_{m-1}s^{m-1} \cdots + b_1 s + b_0)}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0} \tag{1}$$
for $m \le n$, $k_{\mathrm{p}} > 0$, and Hurwitz $D_C(s)$? Explain via root-locus arguments. What property this controller lacks to stabilize the system?

3. Consider now
$$C(s) = \frac{k_{\mathrm{p}}(s + \alpha - \mu\gamma)(s + \mu\gamma)}{(s - \alpha - \gamma)(s + \gamma)} \tag{2}$$
for $k_{\mathrm{p}} > 0$ and $\alpha > \mu\gamma$ (it cancels *stable* pole and zero of $P(s)$ and has a LHP zero at $s = -\alpha + \mu\gamma$ and a RHP pole at $s = \alpha + \gamma$). Under what conditions on k_{p} and α this controller stabilizes the plant? Explain via analyzing roots of the closed-loop characteristic polynomial.

4. Do the same, but using root-locus arguments. Find then k_{p} and α such that the closed-loop system has all its poles at $s = -\gamma$.

5. Simulate the closed-loop system under the the pulse input disturbance $d(t) = \mathbb{1}(t - 0.5) - \mathbb{1}(t - 2.5)$ with $m = 5$ [kg], $M = 10$ [kg], $l = 1$ [m], and $g = 9.8$ [m/sec^2] for the parameters k_{p} and α chosen in the previous item.

6. Simulate the closed-loop system in the case when the standard gravity in the plant model is actually is $g = 9.80665$ [m/sec^2] (affecting γ), whereas the controller is still designed assuming $g = 9.8$ [m/sec^2] (i.e. pole-zero cancellations between the plant and the controller are not exact).