Introduction to Control (00340040)

tutorial 5

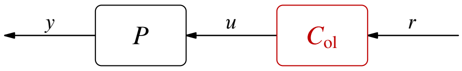


Fig. 1: Open-loop control system

Question 1. Consider the open-loop control system in Fig. 1 for the plant

$$P(s) = \frac{0.2s + 1}{(0.5s + 1)(s^2 + 0.2s + 1)}.$$

1. Can this plant be controlled via the use of a reference model? If it can, what conditions must be met by the reference model to guarantee the internal stability of the control system?

2. Consider a second-order reference model $T_{ref} : r \mapsto y$ with the transfer function

$$T_{ref}(s) = \frac{\omega_n^2}{s^2 + \sqrt{2}\omega_n s + \omega_n^2}$$

   (this is the second-order low-pass Butterworth filter, whose magnitude frequency response satisfies $|T_{ref}(j\omega)| = 1/\sqrt{1 + \omega^4/\omega_n^4}$ and whose bandwidth $\omega_b = \omega_n$). What will be the resulting C_{ol}? Is it admissible?

3. Plot the step responses of y and u and the magnitude bode plots of the reference model and the plant itself for $\omega_n \in \{0.4, 1, 3, 10\}$.

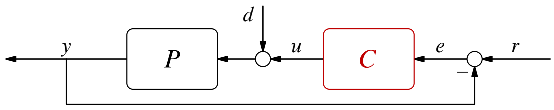


Fig. 2: Unity feedback closed-loop system

Question 2. A plant with the transfer function

$$P(s) = \frac{1}{s(s + 1)(s + 2)}$$

is controlled in the unity feedback scheme with static (proportional) controllers of the form $C(s) = k_p$, see Fig. 2.

1. Derive the four closed-loop transfer functions (the Gang of Four) for this system. What signals in Fig. 2 each of them connects? What is the closed-loop characteristic polynomial? Under what controller gains the closed-loop system is internally stable?

2. Let $k_p \in \{1, 4, 7\}$. Plot the responses of each closed-loop system to a unit step. Explain the differences between the responses for different values of k_p.

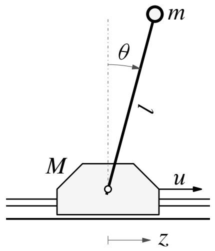


Fig. 3: Inverted pendulum on a cart

Question 3. Consider an inverted pendulum which consists of a point mass m on a mass-less rod of length l installed on a cart of mass M. An external force u is acting on the cart. The equations of motion of this system (see the Linear Systems M course notes) are

$$(M+m)\ddot{z}(t) + ml\ddot{\theta}(t)\cos\theta(t) - ml\dot{\theta}^2(t)\sin\theta(t) = u(t) \tag{1a}$$

$$\ddot{z}(t)\cos\theta(t) + l\ddot{\theta}(t) - g\sin\theta(t) = 0 \tag{1b}$$

where θ is the angle of the pendulum and z is the position of the cart. The parameters are $m = 5$ [kg], $M = 10$ [kg], $l = 1$ [m], and the standard gravity is taken $g = 9.8$ [m/sec^2]

0. Derive the linearized state-space model of the system (in the "up" position) and the transfer function $P(s)$ with u as its input and the car acceleration $y = \ddot{z}$ as its output.

1. The system is controlled in a standard unity feedback closed-loop scheme, like that in Fig. 2. Can it be controlled (that is, stabilized) by the controller

$$C(s) = \frac{10(s^2 - 14.7)}{s^2 + 4s + 11.8} \tag{2}$$

   Check that both via the stability of the closed-loop transfer functions $T(s)$, $S(s)$, $T_{\mathrm{d}}(s)$, and $T_{\mathrm{c}}(s)$ and via the characteristic polynomial of the closed-loop system.

2. Analyze the step responses of the system $r \mapsto y$ under the controller above and no disturbances if the standard gravity is actually $g = 9.80665$ [m/sec^2] ($\approx 0.07\%$ deviation from the assumed g).

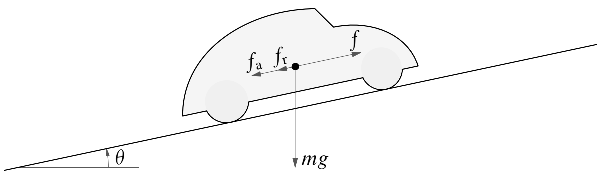


Fig. 4: System for Question 4

Question 4 (self study)**.** Fig. 4 depicts a vehicle of mass $m = 1000$ [kg] driving uphill with the slope $\theta = 12°$. The driving force f generated by the engine is the control signal, whose goal is to maintain the car velocity v at a pre-specified level. The resistance force has three major components: $f_{\mathrm{g}} = mg\sin\theta$, the forces due to gravity; f_a, the aerodynamic drag; and f_r, the forces due to rolling friction. Assuming that the velocity of the car is always positive, the rolling resistance $f_r = mgc_r \cos\theta$, where the rolling resistance coefficient $c_r = 0.01$. The aerodynamic drag is proportional to the square of the speed, i.e. $f_a = \frac{1}{2}\alpha v^2$, where $\alpha \approx 1$ [kg/m] is a constant depending on the density of air, the frontal area of the car, and a shape-dependent aerodynamic drag coefficient. As shown in Tutorial 3, the nonlinear motion equation of this system is

$$\dot{v}(t) = \frac{1}{m}F(t) - \frac{1}{2m}\alpha v^2(t) - g(\sin\theta + c_r\cos\theta)$$

and linearized motion equation around the equilibrium velocity $v_{eq} = 80$ [km/h] $= 200/9 \approx 22.22$ [m/sec] is

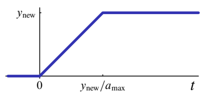
$$\dot{y}(t) = -\frac{\alpha v_{eq}}{m}y(t) + \frac{1}{m}u(t),$$

where the deviation variables $y := v - v_{eq}$ and $u := f - 0.5\alpha v_{eq}^2 - mg(\sin\theta + c_r\cos\theta)$.

1. Consider the unity feedback closed-loop control strategy in which a *proportional* controller $C(s) = k_p$ generates the control signal $u(t)$ from the mismatch between the reference velocity signal $r_v(t)$ and the measured deviation from the equilibrium velocity $y(t)$. Draw the block-diagram of this system. Under what values of k_p the closed-loop system is stable?

2. Consider the reference signal r_v such that

$$r_v(t) = \begin{cases} 0 & \text{if } t \le 0 \\ a_{max}t & \text{if } 0 \le t \le y_{new}/a_{max} \\ y_{new} & \text{if } t \ge y_{new}/a_{max} \end{cases} = \qquad (3)$$



   for the peak acceleration $a_{max} = 0.5$ [m/s^2] and $y_{new} = 10$ [km/h] $= 25/9 \approx 2.78$ [m/sec]. How the choice of k_p affects the steady-state error in general? Simulate the response of the linearized system under k_p's for which the steady-state error is $e_{ss} = |\lim_{t\to\infty} r_v(t) - y(t)| \in \{2, 1, 0.1\}$ [km/h].

3. How does the steady-state error of the previous item change if the road slope changes? Simulate with the change from Tutorial 3, $\bar{\theta} = 13°$, under the controller gains obtained in the previous item. How does it differ from the open-loop results of Tutorial 3?

4. Analyze the nonlinear system with the unity feedback closed-loop controller as in item 1. What is its steady-state response to the reference signal in (3)?

Question 5 (self study)**.** Consider the unity feedback closed-loop system in Fig. 2. Let

$$P(s) = \frac{s+1}{s(s^2+s+1)} \quad \text{and} \quad C(s) = \frac{k(\tau s+1)}{s(s+1)}.$$

Determine and draw the closed-loop stability area in the (τ, k)-plane.