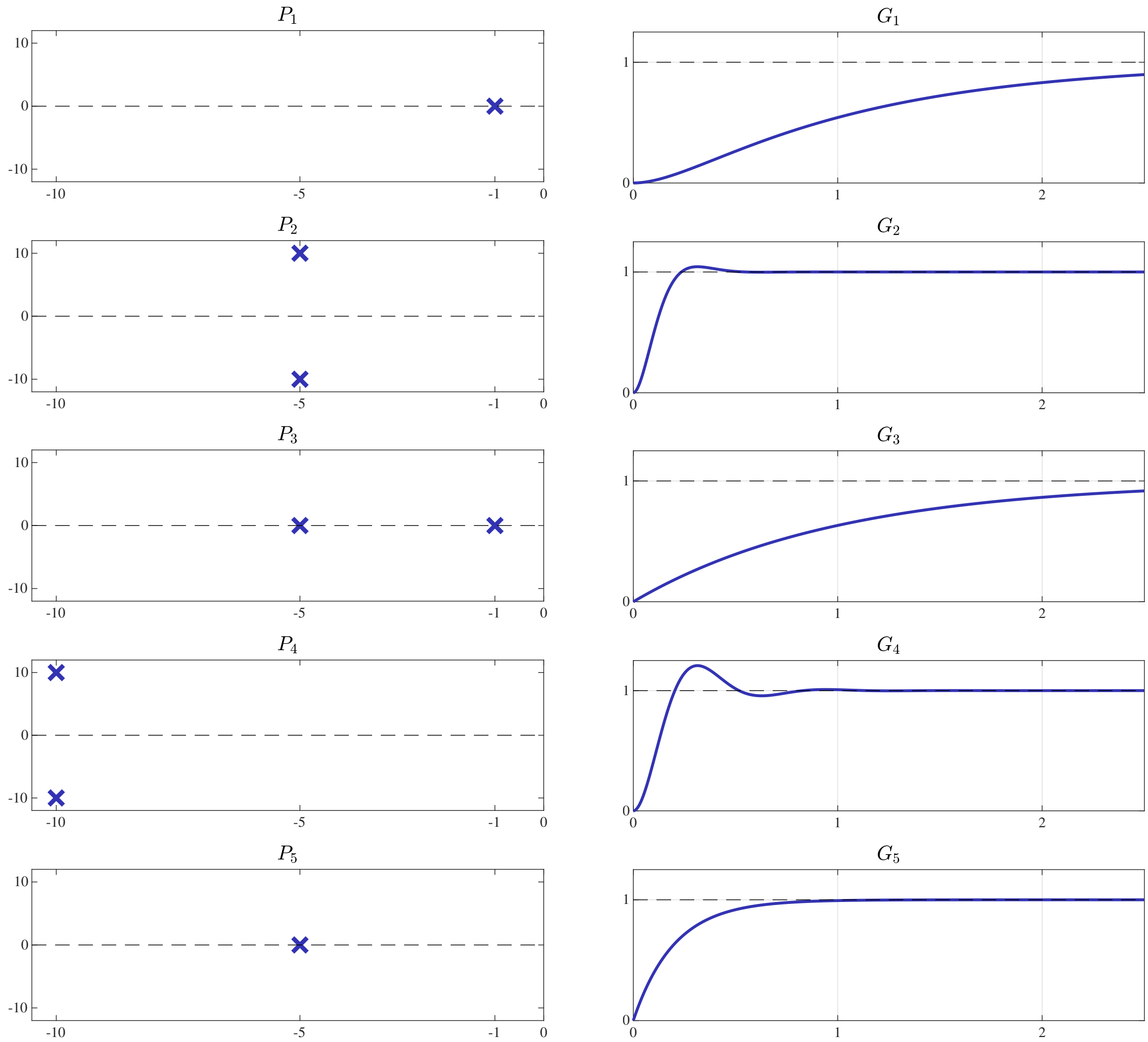
הטכניון – מכון טכנולוגי לישראל, הפקולטה להנדסת מכונות

TECHNION—Israel Institute of Technology, Faculty of Mechanical Engineering

Introduction to Control (00340040)

tutorial 4



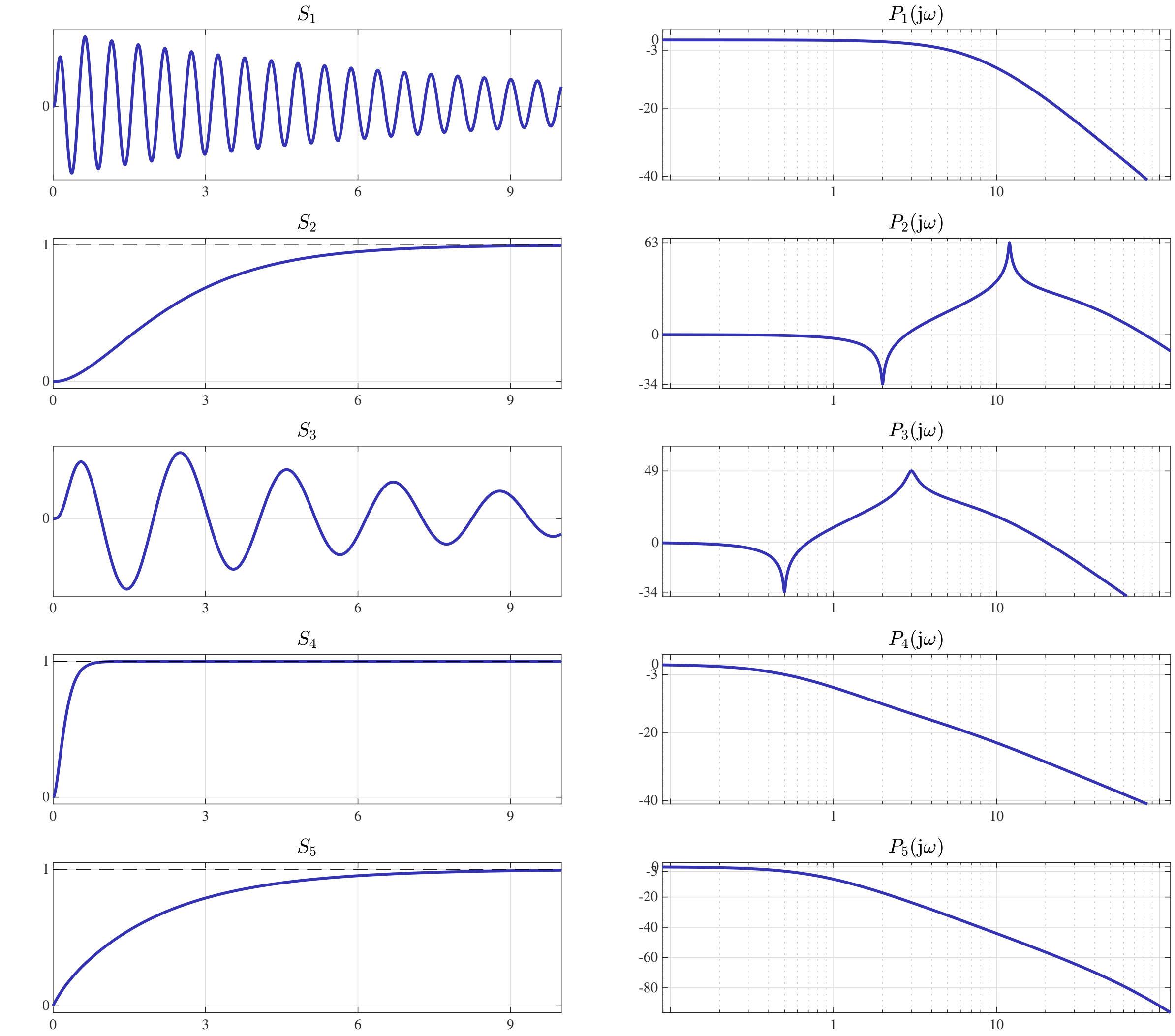
(a) (b)

Fig. 1: Pole-zero maps and step responses for Question 1

Question 1. Match the pole-zero maps given in Fig. 1(a) to the step responses in Fig. 1(b).

Question 2. Match the step responses in Fig. 2(a) to the Bode magnitude plots in Fig. 2(b).

Question 3. Consider the system presented in Fig. 3, which consists of two masses $m = 1$ [kg] connected via a massless pulley and a spring having the spring constant $k = 1$ [N/m] and the damping coefficient $c = 0.1$ [N sec/m]. The contact between the masses causes a friction force which is proportional to the



(a) (b)

Fig. 2: Step responses and Bode magnitude plots for Question 2

velocity, with the coefficient c_m and opposes the motion. The displacement of the right mass is the system input u and the displacement of the left mass is the output y. The system is controlled in a standard open-loop scheme with a controller $C_{\text{ol}} : r \mapsto u$ and controlled response $T_{yr} : r \mapsto y$ for a reference signal r.

1. If the masses do not touch each other (i.e. no friction force is acting between them, with $c_m = 0$), then the plant $P_0 : u \mapsto y$ has the transfer function (cf. Lecture 2)

$$P_0(s) = \frac{cs + k}{ms^2 + cs + k}.$$

   Design the controller C_{ol} for which the controlled responds to a reference signal r has the transfer function

$$T_{yr1}(s) = \frac{1}{\tau s + 1}.$$

   Under what conditions on τ the resulting controller is admissible (i.e. internally stabilizing)?

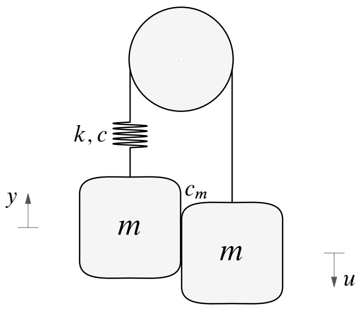


Fig. 3: System for Question 3

2. Plot the step responses of u and y for $\tau \in \{0.1, 0.5, 1, 10\}$ [sec]. Explain trends under decreasing τ using frequency-domain arguments.

3. Now assume that we need to ensure the zero steady-state error between r and y for both $r(t) = \mathbb{1}(t)$ and $r(t) = \sin(\omega_r t + \phi_r)\mathbb{1}(t)$ for a given $\omega_r > 0$ and every $\phi_r \in \mathbb{R}$. Consider the family of controlled transfer functions of the form

$$T_{yr2}(s) = \frac{\alpha_2 s^2 + \alpha_1 \omega_\mathrm{n} s + \alpha_0 \omega_\mathrm{n}^2}{((\beta/\omega_\mathrm{n})s + 1)(s^2 + 2\zeta\omega_\mathrm{n} s + \omega_\mathrm{n}^2)}$$

   for $\omega_\mathrm{n} = 1$, $\zeta = 3$, $\beta = 2$, and some $\alpha_i \in \mathbb{R}$, $i \in \{0, 1, 2\}$, to be chosen to satisfy the steady-state requirements. Is the resulting controller C_ol stabilizing? It it is, calculate α_i for $\omega_r = 1$. Plot the Bode diagram of its frequency response.

4. Now, suppose that the masses touch each other, producing a friction force with friction coefficient $c_m = 0.5$ [N sec/m]. Derive the motion equation and the transfer function of the system $P_{c_m} : u \mapsto y$ in that case.

5. Draw (schematically) the systems step response.

6. Is the controller designed for P_{c_m} under the desired controlled system $T_{\mathrm{ref},1}$ as in item 1 admissible? What requirements must be satisfied by the controlled transfer function in this case to result in an admissible (internally stabilizing) controller?

7. Consider now the design of C_ol for P_{c_m} so that the controlled dynamics have

$$T_{yr3}(s) = \frac{\omega_\mathrm{n}^2 (b_1 s + 1)}{s^2 + 2\zeta\omega_\mathrm{n} s + \omega_\mathrm{n}^2}.$$

   What constraints do we have on b_1? Design the controller C_ol such that the resulting system responds to reference signals r exactly as T_{yr3} for the chosen b_1. Plot the step responses of u and y for $\omega_\mathrm{n} \in \{0.1, 1, 2, 5\}$ and $\zeta = \sqrt{0.5}$. Explain trends under increasing ω_n using frequency-domain and modal arguments.

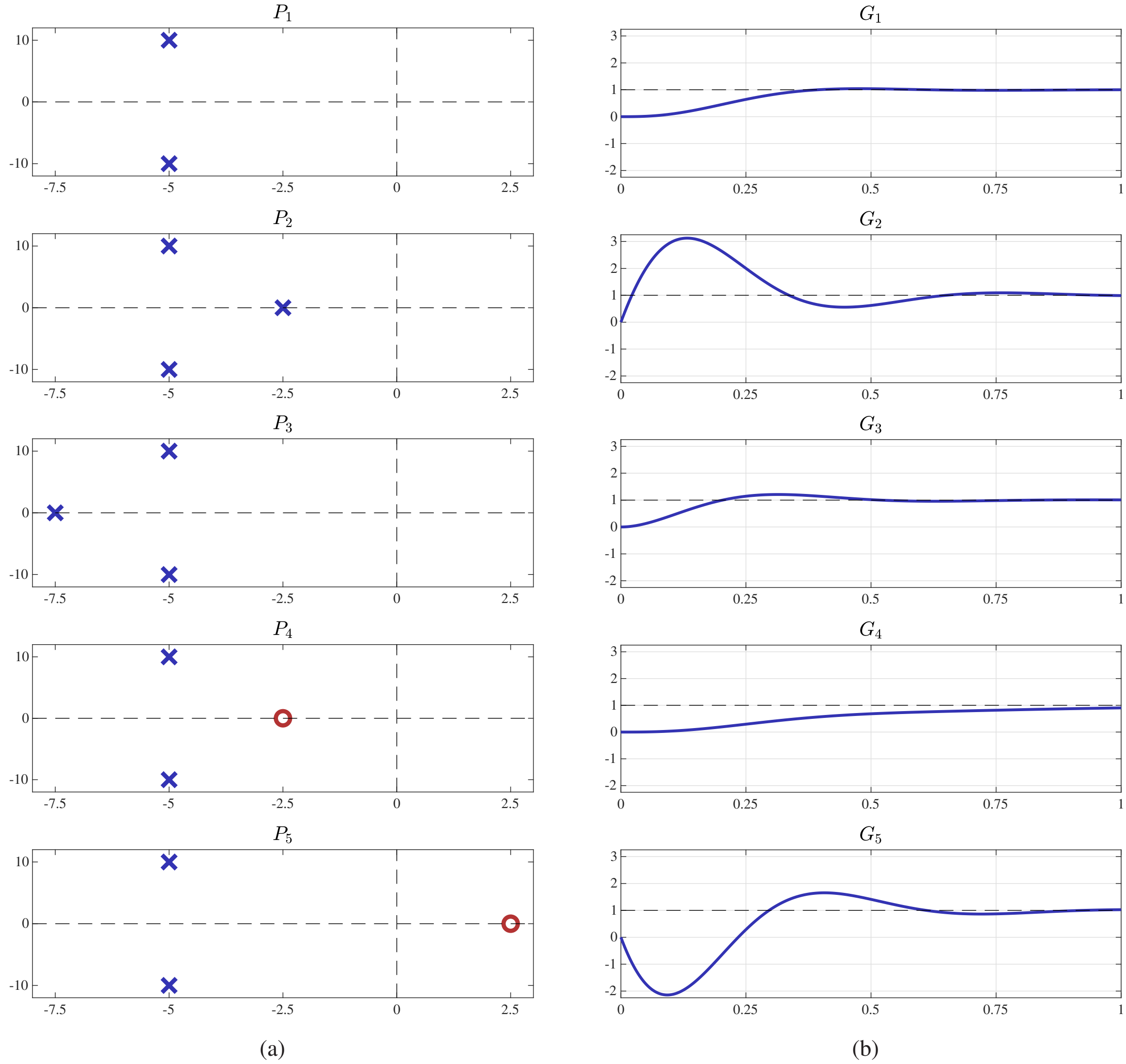


Fig. 4: Pole-zero maps and step responses for Question 4

Question 4 (self study)**.** Consider now a second-order underdamped system, like that given by P_2 in Fig. 1(a) with various modifications. Match the pole-maps given in Fig. 4(a) to the step time-responses in Fig. 4(b).