הטכניון – מכון טכנולוגי לישראל, הפקולטה להנדסת מכונות

TECHNION—Israel Institute of Technology, Faculty of Mechanical Engineering

Introduction to Control (034040)

tutorial 3

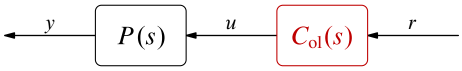


Fig. 1: Open-loop control system

Question 1. Consider the following plants controlled in open loop as illustrated in Fig. 1:

1. $P(s) = \frac{1}{s+1}$
2. $P(s) = \frac{s-2}{s+1}$
3. $P(s) = \frac{s+2}{s-1}$

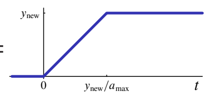
Can these plants be controlled by the controller $C_{\text{ol}} = P^{-1}$?

Question 2. Fig. 2 depicts a vehicle of mass $m = 1000$ [kg] driving uphill with the slope $\theta = 12°$. The driving force f generated by the engine is the control signal, whose goal is to maintain the car velocity v at a prespecified level. The resistance force has three major components: $f_{\text{g}} = mg \sin\theta$, the forces due to gravity; f_{a}, the aerodynamic drag; and f_{r}, the forces due to rolling friction. Assuming that the velocity of the car is always positive, the rolling resistance $f_{\text{r}} = mgc_{\text{r}} \cos\theta$, where the rolling resistance coefficient $c_{\text{r}} = 0.01$. The aerodynamic drag is proportional to the square of the speed, i.e. $f_{\text{a}} = \frac{1}{2}\alpha v^2$, where $\alpha \approx 1$ [kg/m] is a constant depending on the density of air, the frontal area of the car, and a shape-dependent aerodynamic drag coefficient.

1. Derive the equation of motion of the system and linearize it around the speed equilibrium point $v_{\text{eq}} = 80\,[\text{km/h}] = \frac{80\cdot 1000}{3600} = \frac{200}{9} \approx 22.22\,[\text{m/sec}]$ (denote the the control input and the regulated output in terms of deviation variables by u and y, respectively).

2. Consider the open-loop control strategy in which a controller C_{ol} generates the control signal u from a reference velocity signal r_v. Draw the block-diagram of this system. Design C_{ol} by plant inversion. When can this controller be implemented? Simulate the response of the linearized system under

$$r_v(t) = \begin{cases} 0 & \text{if } t \le 0 \\ a_{\max} t & \text{if } 0 \le t \le y_{\text{new}}/a_{\max} \\ y_{\text{new}} & \text{if } t \ge y_{\text{new}}/a_{\max} \end{cases} = \quad (1)$$



for the peak acceleration $a_{\max} = 0.5\,[\text{m/s}^2]$ and $y_{\text{new}} = 10\,[\text{km/h}] = \frac{25}{9} \approx 2.78\,[\text{m/sec}]$.

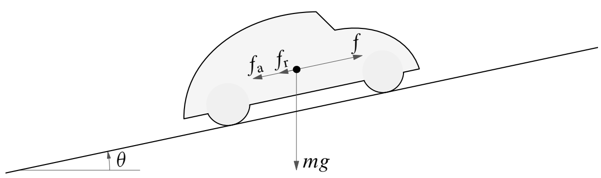


Fig. 2: System for Question 2

3. Assume that the road slope is actually different from the slope at which we modeled the car and designed our controller, viz. it equals $\bar{\theta} = 13°$. Write the new equation of motion in this case and and simulate the linearized response to the saturated ramp in (1). What is the settling time (to the settling level 5%) in this case?

4. To cope with slope uncertainty, a slope sensor can be installed in the vehicle. Suggest an open-loop control strategy, which exploits the information about the actual slope and guarantees that the regulated output $y = r_v$ for all t under any change in θ.

5. Assume now that the slope is as expected, but a passenger weighing $70g$ [N] entered the vehicle, so that its actual vehicle mass becomes $\bar{m} = 1070$ [kg]. Find the actual linearized dynamics and simulate the linearized response to the saturated ramp in (1). What is the settling time (to the settling level 5%) in this case?

6. Analyze the nonlinear system with the open-loop controller as in item 2. What is its steady-state response to the reference signal in (1)?