Introduction to Control (034040)

tutorial 2

Question 1. Fig. 1 depicts a system for controlling the angle $\theta_a(t)$ of an antenna. The antenna, having

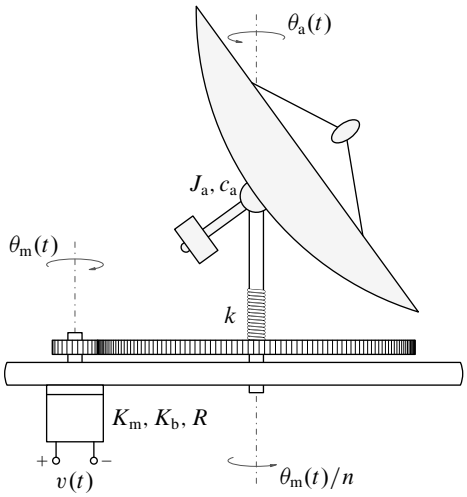

Fig. 1: Antenna actuated via a DC motor

moment of inertia J_a and balanced with respect to the vertical axis, is rotated by a DC motor with torque constant K_m, back emf constant K_b, armature resistance R and negligible inertia and armature inductance. The transmission system comprises two meshing gears, having a ratio of $1 : n$ and negligible inertia, a flexible rod with torsion coefficient (spring constant) k and negligible viscous friction, and a bearing system with viscous friction coefficient c_a. The control input is the voltage v applied to the DC motor.

1. Derive motion equations of the system.
2. Construct the block-diagram of the system and calculate the transfer function of the system $v \mapsto \theta_a$.

Question 2. Consider the magnetic levitation system described in Fig. 2. The electric current i running

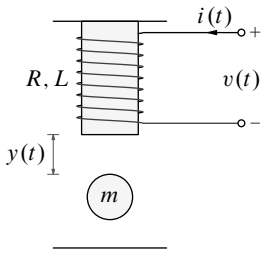

Fig. 2: Magnetic levitation system

through a coil, having resistance R and inductance L, creates a magnetic field, which attracts an iron ball of mass m. The objective is to control the ball position y via the input voltage v. The electromagnetic force applied by the magnetic field to the ball is

$$f_{em}(t) = \alpha \frac{i^2(t)}{y^2(t)}, \tag{1}$$

where $\alpha > 0$ is constant.

1. Write dynamic equations of the systems and its state-space realization.

2. For a given equilibrium position of the ball, $y(t) \equiv y_0$, find the state and input at the equilibrium, linearize the system dynamics around that point, and derive the relation between the deviation variables $\tilde{v}$ and $\tilde{y}$ in the Laplace transform domain (the s-domain).

3. Is the linearized system BIBO stable?

4. Assume that the model and the deviation variables are derived for some m, but the actual mass of the ball is $\bar{m} \neq m$. How does this affect the linearized relation between $\tilde{v}$ and $\tilde{y}$?