Introduction to Control (00340040)

PROJECT

1 Modeling

Consider a DC motor connected with a mechanical load. Let the load comprise a gearhead and a platform connected rigidly. The gearhead increases the torque generated at the motor shaft by its reduction ratio $n_g > 1$, i.e. its action can be described as

$$\tau_g(t) = n_g \tau_m(t),$$

where τ_m and τ_g are the torques generated at the motor shaft and at the gearhead shaft, respectively. The dynamics of the whole mechanical load are

$$J\ddot{\theta}(t) + f\dot{\theta}(t) = \tau_g(t),$$

where J and f are the load moment of inertia and viscous friction, respectively, and θ is the *load* shaft angle related to the motor shaft angle θ_m as $\theta_m = n_g\theta$. The motor is controlled via its armature voltage u.

1. Assuming that the armature inductance is negligible, present the block-diagram of the system. The block-diagram must explicitly contain the signals u, i (armature current), τ_m, τ_g, θ, θ_m, and $\omega := \dot{\theta}$.

2. Derive the transfer functions of the systems $P_\theta : u \mapsto \theta$ and $P_\omega : u \mapsto \omega$. What are the static gain and time constant of P_ω in terms of the problem data?

3. Are P_ω and P_θ stable? How do the answers depend on the parameters?

2 Closed-loop control

We are now in the position to design a controller for our DC motor P_θ to control the load angle. The ultimate goal is to track perfectly a constant reference signal r in steady state, while attenuating a constant disturbance signal d, also in steady state. In all items below assume the following choice of the parameters:

Parameter	K_m [N m/A]	R_a [Ω]	J [kg m^2]	f [N m s/rad]	n_g
Value	0.0242	2.15	0.0047	0.004	4.85

where K_m is the motor torque constant (equals the backemf constant K_m) and R_a is the resistance of the armature circuit.

All questions below are about the configuration in Fig. 1.

4. Consider first a P controller of the form $C(s) = k_p$.

   (a) In what range of k_p the closed-loop system is stable? Use *root-locus arguments* to justify.

   (b) What is the range of k_p for which the steady-state error $e_{ss} := \lim_{t\to\infty}|e(t)| = 0$ under $r = \mathbb{1}$?

   (c) In what range of k_p the damping factors of all closed-loop poles satisfy $\zeta \in (0, 0.69)$? What is the range of overshoot values for the response of θ to $r = \mathbb{1}$ in this range of k_p?

   (d) In what range of k_p the natural frequencies of all closed-loop poles satisfy $\omega_n \geq 2.22$ rad/sec?

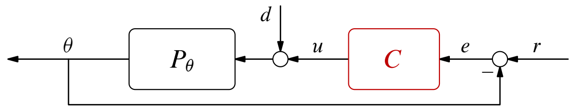

Fig. 1: Closed-loop system for controller design

(e) In what range of k_p the steady-state response to $d = \mathbb{1}$ satisfies $\lim_{t\to\infty}|\theta(t)| \le 1$? What is the minimal attainable overshoot in that range?

5. Consider now a PI controller of the form $C(s) = k_\mathrm{p}\Big(1 + \dfrac{k_\mathrm{i}}{s}\Big)$.

   (a) In what range of k_p and k_i the closed-loop system is stable? Use *root-locus arguments* to justify, again.

   (b) In what range of k_p and k_i the steady-state error $e_\mathrm{ss} := \lim_{t\to\infty}|e(t)| = 0$ under $r = \mathbb{1}$?

   (c) In what range of k_p and k_i the steady-state response to $d = \mathbb{1}$ verifies $\lim_{t\to\infty}|\theta(t)| \le 1$?

   (d) Choose k_p and $k_\mathrm{i} \ge 0.01$ such that the step response of the resulted system $T : r \mapsto \theta$ has OS $\le 15\%$ and $t_\mathrm{s} \le 3$ [sec] under the settling level $\delta = 2\%$.

All answers should be justified. All simulation should be attached in *two* m-files, one for item 4 and another one for item 5.