# Introduction to Control (034040) lecture no. 12

Leonid Mirkin

Faculty of Mechanical Engineering Technion—IIT





[Industrial \(PID\) controllers](#page-2-0)

- [Tuning PID controllers](#page-10-0)
- [PID controller architectures and implementation](#page-33-0)

[2-degrees-of-freedom controller configuration](#page-41-0)

[Concluding remarks](#page-55-0)



### <span id="page-2-0"></span>[Industrial \(PID\) controllers](#page-2-0)

- 
- 



 $- P: C(s) = k_p;$ 



 $- P: C(s) = k_p;$ − PI:

$$
C(s) = k_{p} \left( 1 + \frac{k_{i}}{s} \right) = k_{p} \left( 1 + \frac{1}{\tau_{i}s} \right),
$$

with  $\tau_i = 1/k_i$  called its reset time;

$$
C(s)=k_{\rm p}(1+\tau_{\rm d}s),
$$

$$
C(\mathfrak{s}) = k_p \Big( 1 + \frac{1}{\tau_i \mathfrak{s}} + \tau_d \mathfrak{s} \Big) \quad \text{or} \quad C(\mathfrak{s}) = k_p \Big( 1 + \frac{1}{\tau_i \mathfrak{s}} \Big) \Big( 1 + \tau_d \mathfrak{s} \Big).
$$



 $- P: C(s) = k_p;$ − PI:

$$
C(s) = k_{p} \left( 1 + \frac{k_{i}}{s} \right) = k_{p} \left( 1 + \frac{1}{\tau_{i}s} \right),
$$

with  $\tau_i = 1/k_i$  called its reset time; − PD:

$$
C(s)=k_{p}(1+\tau_{d}s),
$$

with  $\tau_d$  called its derivative time;



 $- P: C(s) = k_p;$ − PI:

$$
C(s) = k_{p} \left( 1 + \frac{k_{i}}{s} \right) = k_{p} \left( 1 + \frac{1}{\tau_{i}s} \right),
$$

with  $\tau_i = 1/k_i$  called its reset time; − PD:

$$
C(s)=k_{p}(1+\tau_{d}s),
$$

with  $\tau_d$  called its derivative time; − PID:

$$
C(s) = k_p \Big( 1 + \frac{1}{\tau_i s} + \tau_d s \Big) \quad \text{or} \quad C(s) = k_p \Big( 1 + \frac{1}{\tau_i s} \Big) \Big( 1 + \tau_d s \Big).
$$

## PID regulators: why

- − Relatively simple structure
	- − (relatively) easy to implement
	- − (relatively) easy to tune

- − Intuitively clear interpretation
	- − the "P"-part exploits current knowledge of system behavior
	- − the "I"-part exploits the past
	- − the "D"-part attempts to exploit prediction of the future

− Vast practical experience

#### Frequency domain interpretation of PI controller



PI controller adds additional phase lag thus reducing stability margins

to prevent this,  $\tau_i$  should be such that  $\omega_c \tau_i \gg 1$ 

#### Frequency domain interpretation of PD controller



PD controller adds a phase lead thus increasing stability margins

to exploit this,  $\tau_d$  should be such that  $\tau_d \omega_c \ll 1$ .

<span id="page-10-0"></span>[Industrial \(PID\) controllers](#page-2-0) [Tuning PID controllers](#page-10-0) [PID architectures](#page-33-0) [2DOF controllers](#page-41-0) [Concluding remarks](#page-55-0)



#### [Tuning PID controllers](#page-10-0)

# Designing PID's

Historically, the design of PID controllers is dubbed their tuning. There are roughly two philosophies here:

explicit model-based: starts with a process model, often of the form

$$
P(s) = \frac{k}{\tau s + 1} e^{-sh}
$$
 or  $P(s) = \frac{k}{(\tau_1 s + 1)(\tau_2 s + 1)} e^{-sh}$ ,

and then chooses  $k_\mathsf{p},\,\tau_\mathsf{i},$  and  $\tau_\mathsf{d}$  according to whatever algorithm;

 $−$  implicit model-based: chooses  $k_p$ ,  $\tau_i$ , and  $\tau_d$  directly from outcomes of a (simple) experiment with the plant according to whatever algorithm

− although frequently regarded as model-free, all such experiments implicitly assume that plant dynamics are "simple," say that the gain and phase are monotonically decaying functions of  $\omega$ 

# Ziegler-Nichols tuning

In 1942 John G. Ziegler & Nathaniel B. Nichols from Taylor Instrument Co. have published a paper with two methods of tuning PID controllers, based on

- 1. a closed-loop experiment with a proportional controller,
- 2. an open-loop step response experiment.

These methods were simple and efficient, soundly outperforming available solutions. They are still relevant today (although may no longer be widely used). We study the first of them below.

















Time, t







Increase  $k_p$  slowly until undamped oscillations arise in y:



Denote the gain for which this happens as  $K_u$  and the oscillation period in steady state as  $T_{\mu}$ .



Increase  $k_p$  slowly until undamped oscillations arise in y:



Denote the gain for which this happens as  $K_u$  and the oscillation period in steady state as  $T_{\mu}$ .

Remark: As a matter of fact,  $T_u = 2\pi/\omega_{\phi}$  and  $K_u = \mu_{\rm g} = 1/|P(j\omega_{\phi})|$ .

## Ziegler-Nichols tuning: rules



<sup>&</sup>lt;sup>1</sup>D. Pessen rule; apparently, ZN rule affected by no longer existent hardware limitations.

### Ziegler-Nichols tuning: rules



These values are to be used

− only as a starting point.

Typically, some (nontrivial) manual tuning of controller parameters required.

 $1D$ . Pessen rule; apparently, ZN rule affected by no longer existent hardware limitations.

### Ziegler-Nichols tuning: example



Polar plot of the plant is:



so we get:  $K_u \approx 4.688$  and  $T_u \approx 3.27$ .

PI controller:  $C_{PI}(s) = 2.11(1 + \frac{1}{2.73s})$ PID controller: PID controller:  $C_{\text{PID}}(s) = 3.28(1 + \frac{1}{1.31s} + 0.49s)$ 





PI controller:  $\omega_c = 1.11$ ,  $\mu_g = 1.79$ ,  $\mu_{ph} = 33.7^\circ$ PID controller:  $\omega_c = 1.55$ ,  $\mu_g = 1.97$ ,  $\mu_{ph} = 36.3^\circ$ 



Time, t



Time, t

## Ziegler-Nichols tuning cum grano salis

Let





It has

 $\omega_{\phi} \approx 0.897$  and  $|P(j\omega_{\phi})| \approx 0.556$ ,

so

 $K_u \approx 1.8$  and  $T_u \approx 7$ .

Hence,

$$
C_{\text{PID}}(s) = 1.08 \left( 1 + \frac{1}{3.5s} + 0.875s \right)
$$

$$
= \frac{0.31(1.75s + 1)^2}{s}.
$$

## Ziegler-Nichols tuning cum grano salis (contd)

The loop is then



which yields an unstable closed-loop system . . .

## Where is it now?

Explicit model-based approaches seem to be dominant nowadays. Arguably, the most widely used plant model is the so-called "first-order+delay," like

$$
P(s) = \frac{k}{\tau s + 1} e^{-sh} \quad \text{or} \quad P(s) = \frac{k}{s} e^{-sh}
$$

for some  $\tau > 0$ . In some cases, when more in

#### Where is it now?

Explicit model-based approaches seem to be dominant nowadays. Arguably, the most widely used plant model is the so-called "first-order+delay," like

$$
P(s) = \frac{k}{\tau s + 1} e^{-sh} \quad \text{or} \quad P(s) = \frac{k}{s} e^{-sh}
$$

for some  $\tau > 0$ . In some cases, when more inertial systems are considered, "second-order+delay" models of the form

$$
P(s) = \frac{k}{(\tau_1 s + 1)(\tau_2 s + 1)} e^{-sh}
$$
 or  $P(s) = \frac{k}{s(\tau s + 1)} e^{-sh}$ 

may be picked.

## Where is it now? (contd)

Typical course of action:

- 1. identify parameters by fitting the model to system response
	- − experiments may be carried out in open-loop or in closed-loop settings
	- may be based on step or frequency response characteristics
- 2. tune  $k_{\sf p},\ \tau_{\sf i},\ \tau_{\sf d}$  to result in a "best" response for a given model
	- − some methods are heuristic lookup tables e.g. the so-called SIMC rule sets

$$
k_{\rm p} = \frac{1}{k} \frac{\tau}{h + \tau_{\rm c}} \quad \text{and} \quad \tau_{\rm i} = \min\{\tau, 4(h + \tau_{\rm c})\},
$$

where  $\tau_c$  is a tuning parameter (the closed-loop dominant time constant)

− others use advanced optimization techniques to tune PID parameters (we may also employ brute force parametric search to minimize whatever cost function . . . )

## First-order+delay richness: example

Let the "actual"

$$
P(s)=\frac{(-0.3s+1)(0.08s+1)}{(2s+1)(s+1)(0.4s+1)(0.2s+1)(0.05s+1)^3}.
$$

It may be approximated by

$$
P_1(s) = \frac{1}{2.48s + 1} e^{-1.58s}
$$



## Second-order+delay richness: example

Let the "actual"

$$
P(s)=\frac{(-0.3s+1)(0.08s+1)}{(2s+1)(s+1)(0.4s+1)(0.2s+1)(0.05s+1)^3}.
$$

Its approximation by

$$
P_2(s)=\frac{1}{(1.19s+1)(1.91s+1)}\,\mathrm{e}^{-0.865s}
$$

is even better:



<span id="page-33-0"></span>[Industrial \(PID\) controllers](#page-2-0) [Tuning PID controllers](#page-10-0) [PID architectures](#page-33-0) [2DOF controllers](#page-41-0) [Concluding remarks](#page-55-0)



[PID controller architectures and implementation](#page-33-0)

## "Natural" architecture



with the control signal

$$
u(t) = k_{p} \Big(e(t) + \frac{1}{\tau_{i}} \int_{0}^{t} e(\theta) d\theta + \tau_{d} \dot{e}(t)\Big).
$$

$$
C(s) = k_p \left( 1 + \frac{1}{\tau_i s} + \tau_d s \right) = \frac{k_p \left( \tau_d \tau_i s^2 + \tau_i s + 1 \right)}{\tau_i s}
$$

### "Natural" architecture



with the control signal

$$
u(t) = k_{p}\Big(e(t) + \frac{1}{\tau_{i}}\int_{0}^{t}e(\theta)d\theta + \tau_{d}\dot{e}(t)\Big).
$$

The transfer function of the controller,

$$
C(s)=k_p\Big(1+\frac{1}{\tau_i s}+\tau_d s\Big)=\frac{k_p(\tau_d\tau_i s^2+\tau_i s+1)}{\tau_i s},
$$

has zeros. It can be verified that unless canceled by plant poles,

controller zeros are zeros of the transfer function  $r \mapsto y$ , i.e. of  $T(s)$ . This might be problematic (zeros, especially dominant, complicate analysis and might contribute to increased overshoot).

## Alternative architecture



with the control signal

$$
u(t) = k_{\mathsf{p}}\Bigl(-y(t) + \frac{1}{\tau_{\mathsf{i}}} \int_0^t e(\theta) \mathsf{d}\theta - \tau_{\mathsf{d}} \dot{y}(t)\Bigr).
$$

$$
T_{c}yr(s) = \frac{P(s) k_{p}/(\tau_{i}s)}{1+P(s)C(s)},
$$

#### Alternative architecture



with the control signal

$$
u(t) = k_{\mathsf{p}}\Bigl(-y(t) + \frac{1}{\tau_{\mathsf{i}}} \int_0^t e(\theta) \mathsf{d}\theta - \tau_{\mathsf{d}} \dot{y}(t)\Bigr).
$$

Now the transfer function of the system  $r \mapsto y$ ,

$$
T_{c}yr(s)=\frac{P(s) k_{p}/(\tau_{i}s)}{1+P(s)C(s)},
$$

has only zeros of the plant as its zeros, which might simplify matters. Note,

− disturbance response is not affected by this change of the architecture.

#### Alternative architecture: example

For example, let

$$
P(s) = \frac{k}{s+a} \quad \text{and} \quad C(s) = k_p \Big( 1 + \frac{1}{\tau_i s} \Big)
$$

Then

$$
T(s) = \frac{k k_{\mathsf{p}}(\tau_{\mathsf{i}} s + 1)}{\tau_{\mathsf{i}} s^2 + \tau_{\mathsf{i}}(a + k k_{\mathsf{p}}) s + k k_{\mathsf{p}}}
$$

has a zero at  $s = -1/\tau_i$  (unless  $\tau_i = 1/a$ ). At the same time,

$$
T_{c}yr(s) = \frac{kk_{\rm p}}{\tau_{\rm i}s^2 + \tau_{\rm i}(a + kk_{\rm p})s + kk_{\rm p}}
$$

is a 2-order transfer function without zeros (hence, easier to understand).

The same tuning, but a different architecture:



Responses to  $r = 1$  with  $C_{PI}$  and  $C_{PID}$ 

Time, t

The disturbance response remains the same.

## Implementing the D part

The non-proper  $\tau_d s$  is normally implemented as

 $\tau_{\sf d} s$  $\overline{\alpha \tau_{\sf d} s + 1}$ 

with sufficiently small  $\alpha$  (typically, 0.05  $\leq \alpha \leq 0.3$ ):





<span id="page-41-0"></span>

#### [2-degrees-of-freedom controller configuration](#page-41-0)

### Open-loop control: architecture



Signals of interest:

$$
y = PC_{ol}r + Pd \quad & \boxed{u = C_{ol}r},
$$

where

- $-$  r is a reference signal (requirements)
- − d is a load disturbance

#### Open-loop control: strategy



Plant inversion with reference model:

$$
C=C_{\rm ol}=P^{-1}T_{\rm ref},
$$

where stable  $T_{ref}$  should endeavor to have...

steady-state  $|1 - T_{ref}(j\omega)| \ll 1$  at  $\omega$ 's where the spectrum of r dominates

- transients  $-$  dominant poles of  $T_{\text{ref}}(s)$  are in "good" regions
	- − sufficiently wide, but not too wide, bandwidth of  $T_{ref}(j\omega)$
	- − no high resonant peaks of  $|T_{ref}(j\omega)|$

### Open-loop control: properties



Open-loop architecture is

- $\circ$  efficient in handling command following requirements
- $\ddot{\psi}$  technically simple both  $T_{\text{ref}}$  and  $C_{\text{ol}}$  are stable
	- − all nonminimum-phase zeros of  $P(s)$  as zeros of  $T_{ref}(s)$
	- − pole excess of  $T_{ref}(s)$  > poles excess of  $P(s)$  unless  $\dot{r}$ ,  $\ddot{r}$ , etc measurable
- $\ddot{\wedge}$  does not help in handling uncertainty
	- − modeling inaccuracies in P
	- − disturbances
- $\ddot{\frown}$  inapplicable if the plant P is unstable

#### Closed-loop control: architecture (unity feedback)



Signals of interest:

$$
y = Tr + T_{d}d - T_{n} \quad \& \quad u = T_{c}r - T_{d} - T_{c}n,
$$

where

$$
T(s) = \frac{P(s)C(s)}{1 + P(s)C(s)}
$$
 complementary sensitivity  
\n
$$
T_d(s) = \frac{P(s)}{1 + P(s)C(s)}
$$
 distributed  
\n
$$
T_c(s) = \frac{C(s)}{1 + P(s)C(s)}
$$
 control sensitivity  
\n
$$
S(s) = 1 - T(s)
$$
 sensitivity density

complementary sensitivity

disturbance sensitivity

control sensitivity

#### Closed-loop control: strategy



Internally stabilizing C should endeavor to have  $\dots$ 

Command following:

- $|1 T(j\omega)| \ll 1$  at  $\omega$ 's where the spectrum of r dominates
- $-$  T(j $\omega$ ) has sufficiently wide, but not too wide, bandwidth
- $-$  T(j $\omega$ ) has no high resonance peaks

Disturbance attenuation:

 $|T_d(j\omega)| \ll 1$  at  $\omega$ 's where the spectrum of d dominates

Noise sensitivity:

 $- |T(j\omega)| \ll 1 \& |T_c(j\omega)| \gg 1$  at  $\omega$ 's where the spectrum of n dominates

#### Closed-loop control: properties



#### Closed-loop architecture is

- ^¨ efficient in stabilizing
- ^¨ efficient in handling command following requirements
- ^¨ efficient in attenuating disturbances
- $\ddot{\frown}$  technically nontrivial
	- − high-low gain tradeoffs
	- crossover region acrobatics

#### Closed-loop control: properties



Closed-loop architecture is

- ^¨ efficient in stabilizing
- ^¨ efficient in handling command following requirements
- ^¨ efficient in attenuating disturbances
- $\ddot{\frown}$  technically nontrivial
	- − high-low gain tradeoffs
	- − crossover region acrobatics

partially because it has to address

 $\ddot{\frown}$  too many intrinsically conflicting goals.

## The best of both worlds

Handling uncertainty:



- − no alternative to feedback
- no alternative architecture (C acts after  $n$  and before  $d$ )

Nominal command following:



- − no advantage of feedback
- − open-loop design is simpler

### The best of both worlds

Handling uncertainty:



- no alternative to feedback
- no alternative architecture (C acts after  $n$  and before  $d$ )

Nominal command following:



- no advantage of feedback
- − open-loop design is simpler

Natural question:

− can we synergize open- and closed-loop architectures?

## Circumvent C in the nominal case

The idea is to

negate the effect of  $C$  if everything behaves expectably.



Expectable behavior is expressed as two requirements to added signals:

1.  $y_{des}$  and  $u_{rea}$  are bounded stability 2.  $y_{des} = Pu_{rea}$  consistency

## Circumvent C in the nominal case

The idea is to

negate the effect of  $C$  if everything behaves expectably.



Expectable behavior is expressed as two requirements to added signals:

1.  $y_{des}$  and  $u_{rea}$  are bounded stability 2.  $V_{\text{des}} = Pu_{\text{req}}$  consistency

In this case (remember,  $T_c P = T$  and  $S + T = 1$ )

 $u = u_{\text{rea}} + C(y_{\text{des}} - Pu) \iff u = Su_{\text{rea}} + T_c y_{\text{des}} = u_{\text{rea}}$ 

and  $y = Pu = y_{des}$ , regardless C (provided it is stabilizing, of course).

2DOF architecture (when a reference model is used)

If we take  $y_{\sf des} = \mathcal{T}_{\sf ref} r$  and  $u_{\sf req} = \mathcal{C}_{\sf ol} r$  for  $\mathcal{C}_{\sf ol} = P^{-1} \mathcal{T}_{\sf ref}$  (consistent), then



#### and signals of interest are

$$
y = T_{ref}r + T_{d}d - T_{n} \quad \& \quad u = C_{ol}r - T_{d} - T_{c}n,
$$

- 
- 

2DOF architecture (when a reference model is used)

If we take  $y_{\sf des} = \mathcal{T}_{\sf ref} r$  and  $u_{\sf req} = \mathcal{C}_{\sf ol} r$  for  $\mathcal{C}_{\sf ol} = P^{-1} \mathcal{T}_{\sf ref}$  (consistent), then



and signals of interest are

$$
\boxed{y = T_{\text{ref}}r + T_{\text{d}}d - Tn} \quad \& \quad \boxed{u = C_{\text{ol}}r - Td - T_{\text{c}}n},
$$

This controller blends open- and closed-loop controller architectures, with a complete separation of

- **nominal command response** shaped by  $C_{0}$ standard open-loop design
- **stabilization and handling uncertainty** shaped by C standard feedback design, just w/o taking the command response into account



<span id="page-55-0"></span>

[Concluding remarks](#page-55-0)

## Summary

Learned in this course,

flavor of basic control ideas,

so you could communicate with control engineers in their native language.

- 
- 
- 
- 
- 
- 

# Summary

Learned in this course,

− flavor of basic control ideas,

so you could communicate with control engineers in their native language.

A long and winding road to become a control engineer  $/R&D$  yourselves, 035188 Control Theory

- 035036 Control Systems Design
- 034406 Advanced Control Laboratory
- 036709 Sampled-Data Control Systems
- 036012 Linear Control Systems
- 036050 Nonlinear Control Systems
- 036013 Process Optimization .

. .