Introduction to Control (00340040) lecture no. 7

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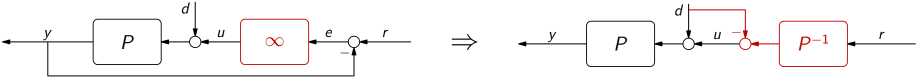
The marvels of high-gain feedback (from Lecture 5)



and

$$T_c \to P^{-1} : r \mapsto u, \quad T \to 1 : r \mapsto y, \quad T_d \to 0 : d \mapsto y$$

i.e. high-gain feedback

- can invert the plant w/o knowing its model
- can compensate disturbance w/o measuring it directly

But

- is it feasible (stabilizing)?

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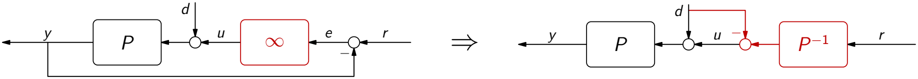

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Outline

High-gain feedback limitations: root locus insight

Back to reality

Steady-state performance of closed-loop systems: conditions for $e_{ss} = 0$

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Steady-state performance of closed-loop systems: conditions for $e_{ss} = 0$

Root-locus behavior as $k \to \infty$

The root locus form

$$-\frac{1}{k} = G_k(s),$$

where $G_k(s)$ has n poles and $m \le n$ (finite) zeros. In this case

- m loci end up at (finite) zeros of $G_k(s)$
- $n - m$ loci end up at infinity along asymptotes centered at

$$\sigma_c = \frac{\sum_{i=1}^{n} p_i - \sum_{i=1}^{m} z_i}{n - m}$$

and directed with angles

$$\phi_i = \frac{\arg b_m - \pi + 2\pi i}{n - m}, \qquad i = 0, 1, \ldots, n - m - 1$$

where b_m is known as the high-frequency gain[1] of G_k.

[1] As high frequencies $|G_k(j\omega)|$ behaves like the gain frequency response of b_m/s^{n-m}.

Strictly proper $G_k(s)$ with negative high-frequency gain

Let

- $n > m$
- $b_m < 0$

Then there always is an asymptote with $\phi_i = 0$, i.e.

- at least one closed-loop pole is in the RHP as $k \to \infty$,

implying that

- high-gain feedback is impossible in this case.

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Nonminimum-phase $G_k(s)$

Let

- at least one (finite) zero of $G_k(s)$ be in $\bar{\mathbb{C}}_0 := \{s \in \mathbb{C} \mid \operatorname{Re} s \geq 0\}$

In this case

- at least one locus ends up[2] in the RHP,

implying that

- high-gain feedback is impossible in this case.

[2]Even if all NMP zeros of $G_k(s)$ are on the jω-axis, closed-loop poles approach the RHP as the gain grows, which is also unacceptable.

Nonminimum-phase $G_k(s)$

Let

– at least one (finite) zero of $G_k(s)$ be in $\bar{\mathbb{C}}_0 := \{s \in \mathbb{C} \mid \operatorname{Re} s \geq 0\}$

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$G_k(s)$ with pole excess more than 2

Let

- $n - m > 2$
- $b_m > 0$

Then one of the asymptotes has $\phi_i = \frac{\pi}{n-m} < \frac{\pi}{2}$, i.e.

- at least two (because of symmetry) closed-loop poles are in the RHP as $k \to \infty$,

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Minimum-phase $G_k(s)$ with pole excess 2

Let

- $n - m = 2$
- $b_m > 0$
- all (finite) zeros of $G_k(s)$ be in $\operatorname{Re} s < 0$

Then $n - 2$ loci end up in the LHP (at stable zeros) and the others go to

$$\frac{\sum_{i=1}^{n} p_i - \sum_{i=1}^{n-2} z_i}{2} \pm \mathrm{j}\infty.$$

implying that

- high-gain feedback is possible in this case iff

$$\sum_{i=1}^{n} p_i < \sum_{i=1}^{n-2} z_i \quad \Longleftrightarrow \quad \frac{a_{n-1}}{a_n} > \frac{b_{n-3}}{b_{n-2}}.$$

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[2] Loosely speaking, this implies that poles should be "more stable" than zeros.

Minimum-phase $G_k(s)$ with pole excess 1

Let

- $n - m = 1$
- $b_m > 0$
- all (finite) zeros of $G_k(s)$ be in $\operatorname{Re} s < 0$

Then $n-1$ loci end up in the LHP (at stable zeros) and the last one goes to $-\infty$ along the real axis, implying that

- high-gain feedback is always possible in this case.

Minimum-phase $G_k(s)$ with pole excess 1

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- $n - m = 1$
- $b_m > 0$
- all (finite) zeros of $G_k(s)$ be in $\operatorname{Re} s < 0$

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Minimum-phase $G_k(s)$ with pole excess 0

Let

- $n - m = 0$
- all (finite) zeros of $G_k(s)$ be in $\operatorname{Re} s < 0$

Then all loci end up in the LHP (at stable zeros), implying that

- high-gain feedback is always possible in this case.

Summary

Systems, for which high-gain feedback can be applied:

1. minimum-phase and $m = n$
2. minimum-phase, $m = n - 1$, and $b_{n-1} > 0$
3. minimum-phase, $m = n - 2$, $b_{n-2} > 0$, and $\frac{a_{n-1}}{a_n} > \frac{b_{n-3}}{b_{n-2}}$
   (although in this case we'll have a pair of lightly damped poles)

MP systems with a pole excess of at most 1 are the classes of systems

– easiest to control by feedback.

The problem is that such systems

– virtually do not exist in real world applications. . . .

We may have such loops if we measure enough derivatives of the output y (equivalent to using non-proper controller). Yet measuring derivatives is

– prone to severe high-frequency noise,

which, as we know from Lecture 5, makes high-gain feedback unaffordable.

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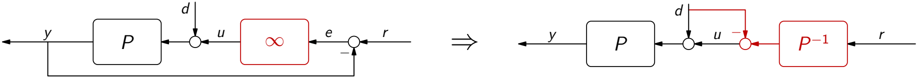
Steady-state performance of closed-loop systems: conditions for $e_{ss} = 0$

The marvels of high-gain feedback (contd)



This control law, which renders

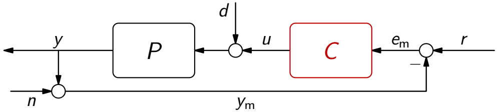
$$T_c \to P^{-1}, \quad T \to 1, \quad T_d \to 0$$

w/o the need to know the plant model, is hardly ever feasible because of

- stability constraints
- measurement imperfections
- implementation limitations

  normally, $C(s)$ must be proper; too large coefficients $\implies$ numerical errors
- ...

What can we do then ?



To understand what can be done, let us try to see how feedback affects

- steady-state errors
- transient behavior

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High-gain feedback limitations: root locus insight

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Steady-state performance of closed-loop systems: conditions for $e_{ss} = 0$

Steady-state error (from Lecture 3)

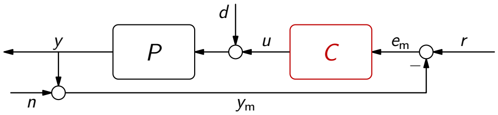
If

$$e = T_{ew} w$$

for some stable T_{ew}, then

$$e_{ss} = \begin{cases} |T_{ew}(j\omega)| & \text{if } w(t) = \sin(\omega t + \phi)\mathbb{1}(t) \\ |T'_{ew}(0)| & \text{if } w(t) = \text{ramp}(t) \text{ and } T_{ew}(0) = 0 \end{cases}$$

Closed-loop relations (from Lecture 5)



We know that

$$y = Tr + T_d d - Tn$$

$$\Downarrow$$

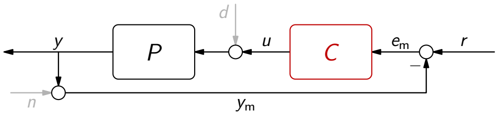
$$e = r - y = Sr - T_d d + Tn,$$

where

$$T(s) = \frac{P(s)C(s)}{1 + P(s)C(s)}, \quad T_d(s) = \frac{P(s)}{1 + P(s)C(s)}, \quad S(s) = \frac{1}{1 + P(s)C(s)}$$

(mind that $S + T = 1$).

e_{ss} for $r(t) = \sin(\omega t + \phi)\mathbb{1}(t)$



In this case

$$e_{ss} = |S(j\omega)| = \frac{1}{|1 + P(j\omega)C(j\omega)|}$$

provided the system is (internally) stable, of course. Thus

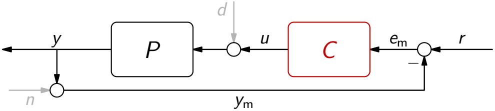
- $e_{ss} = 0 \iff |P(j\omega)C(j\omega)| = \infty \Longleftarrow |P(j\omega)| = \infty \vee |C(j\omega)| = \infty$.

Note that this is a requirement

- only on the gain of PC at one frequency, ω,

and does not impose any requirements on the gain at other frequencies.

e_{ss} for $r(t) = \sin(\omega t + \phi)\mathbb{1}(t)$



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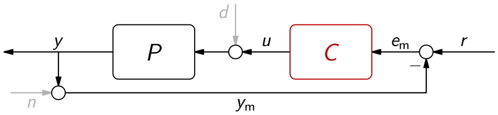

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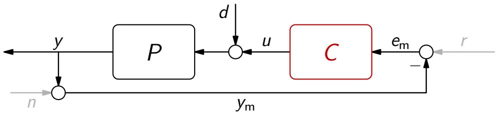

In this case

$$e_{ss} = |-T_d(j\omega)| = \left|\frac{P(j\omega)}{1 + P(j\omega)C(j\omega)}\right| = \frac{1}{|1/P(j\omega) + C(j\omega)|}$$

provided the system is (internally) stable, of course. Thus

– $e_{ss} = 0 \iff$ either $P(j\omega) = 0$ or $|C(j\omega)| = \infty$.

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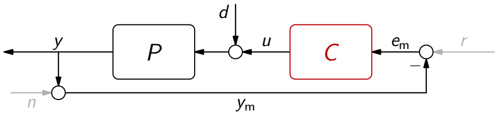

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- $e_{ss} = 0 \iff$ either[3] $P(j\omega) = 0$ or $|C(j\omega)| = \infty$.

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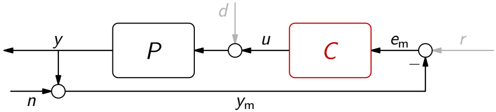
and does not impose any requirements on the gains at other frequencies.

[3] In that case even open-loop control does the job, since the plant filters out harmonic component of d with the frequency ω irrespective of the controller.

e_{ss} for $n(t) = \sin(\omega t + \phi)\mathbb{1}(t)$



In this case

$$e_{ss} = |T(j\omega)| = \left| \frac{P(j\omega)C(j\omega)}{1 + P(j\omega)C(j\omega)} \right|$$

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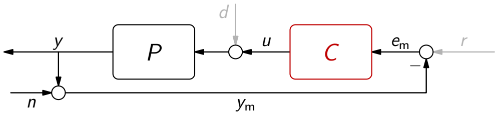

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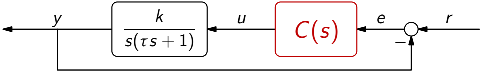
DC motor: e_{ss} for $r(t) = \mathbb{1}(t)$



In this case the plant has a pole at the origin (an integrator), so static loop gain

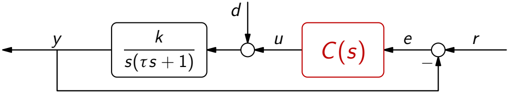
$$|P(0)C(0)| = \lim_{s\to 0}\left|\frac{kC(s)}{s(\tau s+1)}\right| = \infty$$

provided $C(0) \neq 0$ (true whenever C stabilizes the system, right?). Thus

- $e_{ss} = 0$ iff C is stabilizing,

which, for example, true with $C(s) = k_p$ (P controller) for all $k_p > 0$.

DC motor: e_{ss} for $d(t) = \mathbb{1}(t)$



In this case

$$e_{ss} = \frac{1}{|1/P(0) + C(0)|} = \frac{1}{|C(0)|},$$

so that the integral action in P does not help. What could help is an

– integrator in C,

which indeed guarantees that $|C(0)| = \infty$.

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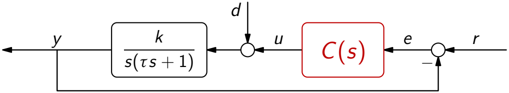

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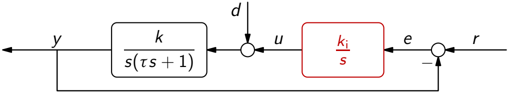


This control law acting as

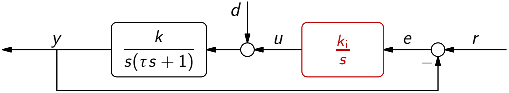
$$u(t) = k_i \int_0^t e(\theta) d\theta$$

is called the integral controller (I controller). This unstable controller allows

- $u(t) \not\to 0$ even if $e(t) \to 0$,

which is necessary for counteracting constant disturbances.

DC motor: e_{ss} for $d(t) = \mathbb{1}(t)$ with I controller



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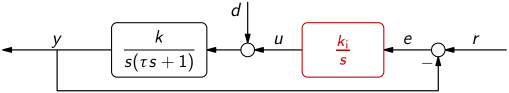
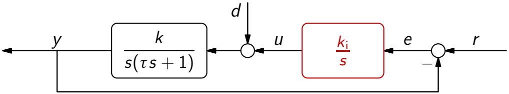
DC motor: e_{ss} for $d(t) = \mathbb{1}(t)$ with I controller (contd)



The root-locus form with respect to k_i is $-\frac{1}{k_i} = \frac{k}{s^2(\tau s+1)}$, so that we have:

which is never stable.

DC motor: e_{ss} for $d(t) = \mathbb{1}(t)$ with I controller (contd)



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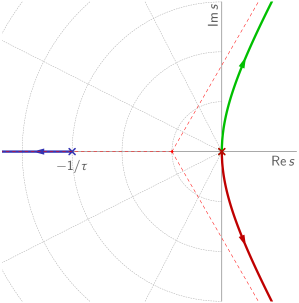

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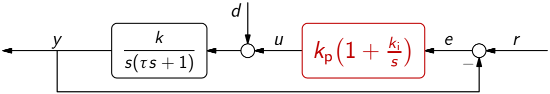
DC motor: e_{ss} for $d(t) = \mathbb{1}(t)$ with PI controller



This control law acting as

$$u(t) = k_p\left(e(t) + k_i \int_0^t e(\theta)\mathrm{d}\theta\right)$$

is called the proportional-integral controller (PI controller). It also allows

– $u(t) \not\to 0$ even if $e(t) \to 0$,

but has an additional degree of freedom, a zero at $s = -k_i$:

$$C(s) = k_p\left(1 + \frac{k_i}{s}\right) = \frac{k_p(s+k_i)}{s},$$

which can be used to stabilize the system.

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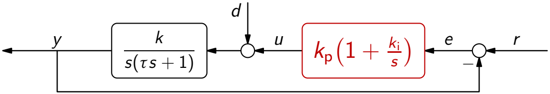

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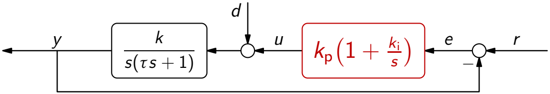

The root locus form with respect to k_p is $-\frac{1}{k_p} = \frac{k(s+k_i)}{s^2(\tau s+1)}$, so that we have:

which may be stabilized iff $0 < \tau k_i < 1$ (as in this case $\sigma_c = \frac{\tau k_i - 1}{2\tau}$).

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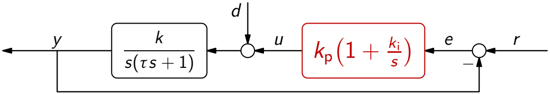

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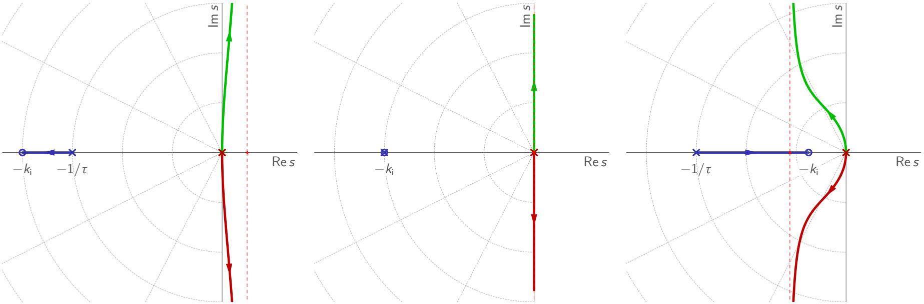

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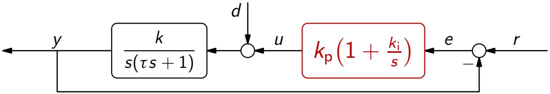

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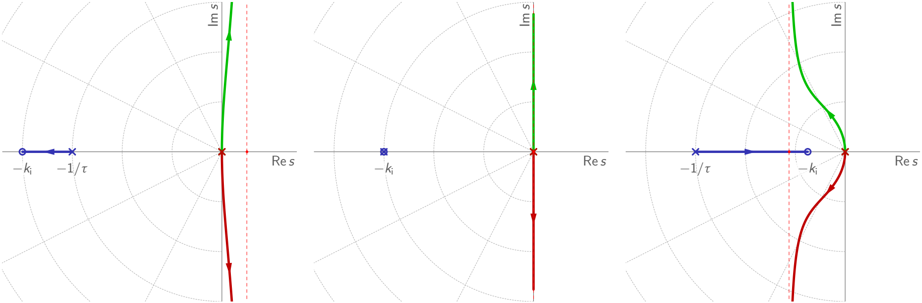

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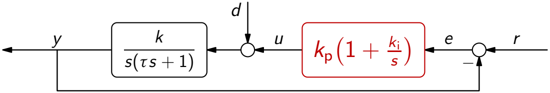
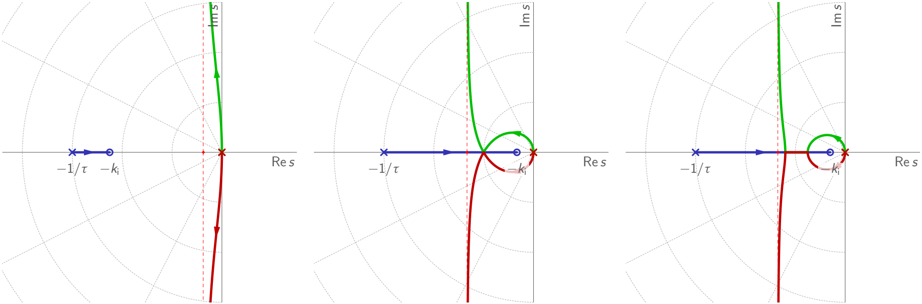
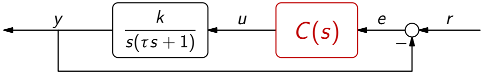


Root loci for various $k_i \in (0, 1/\tau)$ may take exotic forms:



DC motor: e_{ss} for $r(t) = \mathrm{ramp}(t)$



In this case

$$e_{ss} = \left|\lim_{s\to 0} sS(s)\frac{1}{s^2}\right| = \lim_{s\to 0}\frac{1}{|s(1+P(s)C(s))|} = \lim_{s\to 0}\frac{1}{|s+kC(s)/(\tau s+1)|}$$

As $C(0) \neq 0$ for any stabilizing C, we have that

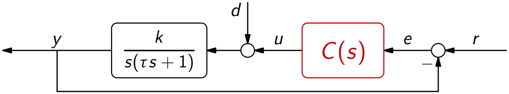
$$e_{ss} = \frac{1}{k|C(0)|}.$$

Thus, the larger $|C(0)|$ is, the smaller e_{ss} is and then

- $e_{ss} = 0$ only if $|C(0)| = \infty$,

which again requires an integral action in C (e.g. PI).

DC motor: e_{ss} for $d(t) = \text{ramp}(t)$



Now,

$$e_{ss} = \left| \lim_{s\to 0} sT_d(s)\frac{1}{s^2} \right| = \lim_{s\to 0} \frac{|P(s)|}{|s(1+P(s)C(s))|} = \lim_{s\to 0} \frac{k}{|s^2(\tau s+1)+ksC(s)|}$$

so that

- if $C(0)$ is finite, then $e_{ss} = \infty$.

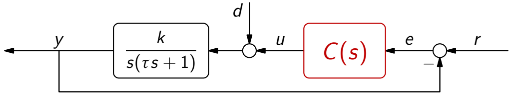
Thus we need an integrator in C just to keep e_{ss} bounded, in which case

$$e_{ss} = \lim_{s\to 0} \frac{1}{|sC(s)|}$$

and

- $e_{ss} = 0$ only if $C(s)$ has at least 2 poles at the origin (double integrator).

DC motor: e_{ss} for $r(t)/d(t) = \sin(\omega t + \phi)\mathbb{1}(t)$, $\omega > 0$



Because

$$|P(\mathrm{j}\omega)| = \left|\frac{k}{\mathrm{j}\omega(\mathrm{j}\omega\tau + 1)}\right| = \frac{k}{\omega\sqrt{1+\tau^2\omega^2}} \neq \infty$$

we have that

– $e_{ss} = 0 \iff |C(\mathrm{j}\omega)| = \infty$.

The latter requires

– poles of $C(s)$ at $\pm\mathrm{j}\omega$,

like

$$C(s) = \frac{b_2 s^2 + b_1 s + b_0}{s^2 + \omega^2}.$$

Try to find stabilizing b_i and show that $b_2 \neq 0$ is necessary for stability.

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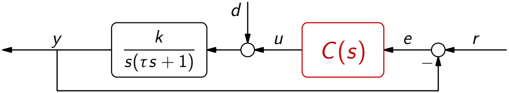

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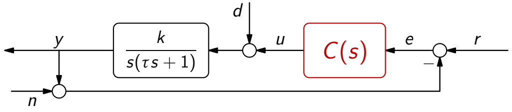


Because

$$P(j\omega) = \frac{k}{j\omega(j\omega\tau + 1)} = \frac{k}{\omega\sqrt{1+\tau^2\omega^2}} \neq 0$$

we have that

- $e_{ss} = 0 \iff C(j\omega) = 0.$

The latter requires

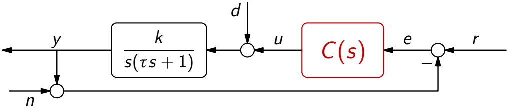
- zeros of $C(s)$ at $\pm j\omega$,

like

$$C(s) = \frac{s^2+\omega^2}{a_2 s^2 + a_1 s + a_0}$$

(known as notch). Try to find stabilizing a_i.

DC motor: e_{ss} for $n(t) = \sin(\omega t + \phi)\mathbb{1}(t)$, $\omega > 0$



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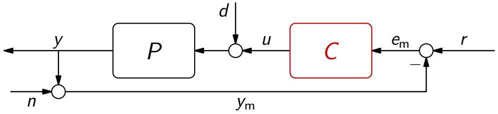
(known as notch). Try to find stabilizing a_i.

Summary



Zero steady-state errors to

- $r(t) = \sin(\omega t + \phi)\mathbb{1}(t)$ requires poles at $\pm j\omega$ in $P(s)C(s)$
- $d(t) = \sin(\omega t + \phi)\mathbb{1}(t)$ requires poles at $\pm j\omega$ in $C(s)$
- $r(t) = \mathbb{1}(t)$ requires an integrator in PC
- $d(t) = \mathbb{1}(t)$ requires an integrator in C
- $r(t) = \text{ramp}(t)$ requires a double integrator in PC
- $d(t) = \text{ramp}(t)$ requires a double integrator in C

which are infinite gains at isolated frequencies. Zero steady-state error to

- $n(t) = \sin(\omega t + \phi)\mathbb{1}(t)$ requires zeros at $\pm j\omega$ in $P(s)C(s)$

which is zero gain at isolated frequencies.