



CONTROL OF MIMO SYSTEMS (00860289)

WINTER 2026/27

HOMEWORK 1

(submission deadline: 13/11/2026, 14:00; do make an effort to be concise, clear, and accurate)

Problem 1 (40pt). Consider an LTI plant $P : u \mapsto y$ having the transfer function

$$P(s) = \frac{(s-1)(s+0.9)}{(s-2)(s+1.8)}.$$

Design an *internally stabilizing* LTI controller *having an integral action and strictly proper transfer function* for it, by whatever method (except AI-generated controllers). Explain, briefly, the logic of your design. Simulate the response of the resulted system (both the output and the control signal) to a step load disturbance for the setup in Fig. 1.4(c) in the Lecture Notes and present those plots. Submit also the stand-alone *.m file producing those plots. Don't use Simulink, the step response of LTI systems can be simulated in Matlab using the `step` command, and don't use direct multiplications / divisions of systems in Matlab (use the `feedback` command to produce closed-loop systems).

Problem 2 (20pt). Prove, using properties of the inner product (§§A.1.2 and A.2.3 in the Lecture Notes), that $A'A \geq 0$ for all $A \in \mathbb{C}^{p \times m}$. What are conditions on A under which $A'A > 0$ in terms of structural properties (i.e. Im or ker) of A (not of A')?

Problem 3 (20pt). A set $\mathcal{S}$ is said to be *convex* if $\lambda x_1 + (1-\lambda)x_2 \in \mathcal{S}$ for all $x_1, x_2 \in \mathcal{S}$ and all $\lambda \in [0, 1]$. Prove that the unit ball $\mathcal{B}_{\mathcal{V}}$ in any normed space $\mathcal{V}$ is convex.

Problem 4 (20pt). Consider the vector space of real polynomials of degree at most 2:

$$\mathbb{P}_2 := \{p : \mathbb{R} \mapsto \mathbb{R} : p(x) = a_0 + a_1x + a_2x^2\}.$$

1. Suggest a basis on this space (prove that it indeed satisfies all requirements). What is the dimension of $\mathbb{P}_2$?
2. Let $\mathcal{D} : \mathbb{P}_2 \mapsto \mathbb{P}_2$ be the differentiation operator ($\mathcal{D}p = \frac{dp(x)}{dx}$). What is the system matrix of $\mathcal{D}$ for the basis chosen in the first item? Is $\mathcal{D}$ invertible? What are Im $\mathcal{D}$, ker $\mathcal{D}$, and rank($\mathcal{D}$)?