Control of Dead-Time Systems
A New Look at Some Old Ideas

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Dead-time systems: single delay case

Here:

- \( P(s) \) – (finite-dimensional) generalized plant
- \( K(s) \) – controller
- \( e^{-sh} \) – loop delay
Dead-time systems: multiple delay case

where the delay operators $\Lambda_y$ and $\Lambda_u$ are of the form

$$\Lambda(s) = \begin{bmatrix} e^{-h_1 s} I_{n_1} & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ & & & e^{-h_r s} I_{n_r} \end{bmatrix}.$$
Why dead-time systems?

1. Delays in physical processes
   - transport delays
   - computational delays
   - ...

Delays in physical processes
Why dead-time systems?

1. Delays in physical processes
   - transport delays
   - computational delays
   - . . .

2. Compact/economical approximations of complex dynamics
Example 1: heating a can (Zwart & Bontsema, 1997)

Transfer function of a heated can (derived from PDE model):

\[
G(s) = \frac{1}{J_0\left(\frac{s}{\alpha R}\right)} + \sum_{m=1}^{\infty} \frac{2}{\lambda_m R} J_1(\lambda_m R) \frac{s}{s + \alpha \lambda_m^2} \frac{1}{\cosh(\sqrt{\frac{s}{\alpha} + \lambda_m^2 \cdot \frac{1}{2}})}.
\]

Its approximation by \(G_2(s) = \frac{1}{(\tau_1 s + 1)(\tau_2 s + 1)}e^{-s\tau}\) is reasonably accurate:
Example 2: torsion of a rod (Raskin & Halevi, 1999)

Transfer function of a free-free uniform rod at distance \( x \) from actuator:

\[
G(s) = \frac{k}{s} \frac{e^{-xhs} + e^{-(2-x)hs}}{1 - e^{-2hs}}, \quad 0 \leq x \leq 1.
\]

Its approximation by \( G_r(s)e^{-xhs} \) does capture high-frequency phase lag:
Why dead-time systems? (contd)

1. Delays in physical processes
   - transport delays
   - computational delays
   - ...

2. Compact/economical approximations of complex dynamics
   - widely appreciated in industry (process control, water distribution control, ...)
     (i.e., E. E. Makovski’s model of water flow in channels; instead of St. Venant eqns)
   - makes sense only if corresponding design tools resulting in conceptually clear
     and numerically reliable controllers are available
Outline

1. Dead-time compensation: Smith predictor and its generalizations

2. Some delay-oriented techniques:
   - loop shifting
   - factorization & MZ trick
   - extraction
   - ...

3. Controller structures and their interpretations

4. Multiple-delay generalizations (example)

5. Concluding remarks
Control of dead-time systems

The presence of loop delays

😊 imposes limitations on the achievable performance,
- controller has “outdated” information
- control action cannot be applied “in time”

😊 might considerably complicate controller design
- dead-time systems are infinite dimensional
Control of DT systems: finite-dimensionalization

✗ plant rationalization
  ➔ rational approximation of $P e^{-sh}$
  ➔ treating delay as uncertainty (robustness embedding)

✗ design of finite-dimensional controllers
  ➔ Lyapunov-based techniques (LMI embedding, etc)
  ➔ pole-placement techniques

😊 finite-dimensional treatment and finite-dimensional controllers

😊 the curse of conservatism

😊 delay structure is typically lost

\(^a\text{Much overlapping between these approaches (even more than conventionally thought).}\)
Control of DT systems: inf.-dimensional embedding

DT systems form one of the simplest classes of distributed-par. models

✗ time-domain methods
   see, e.g., (Curtain & Zwart, 1995)

✗ frequency-domain methods
   see, e.g., (Curtain & Zwart, 1995), (Foias, Özbay, & Tannenbaum, 1996)

😊 precise and rigorous treatment

😊 nontrivial implementation
   (result in infinite-dimensional structureless controllers)

😊 delay structure might be dissolved
Smith controller


- $\tilde{C}$ called primary controller

- $P(1 - e^{-sh})$ called Smith predictor or dead-time compensator (DTC)
  
  (when $r = d = 0$, input of $\tilde{C}$ is $Pu$, which is a “prediction” of $y = Pe^{-sh}u$)

The underlying idea is to

- add internal feedback into controller to pull delay out of the loop.
Smith controller

Closed-loop transfer functions:

\[ T_{yr} = \frac{P\tilde{C}}{1 + P\tilde{C}} e^{-sh} \quad \text{and} \quad T_{yd} = \frac{1 + P\tilde{C}(1 - e^{-sh})}{1 + P\tilde{C}} Pe^{-sh}. \]

-characteristic equation \( 1 + P\tilde{C} = 0 \) contains no exponential terms
Two-stage design philosophy:

1. design $\tilde{C}$ for delay-free plant $P$
2. implement $\tilde{C}$ with DTC loop

Important points:

- design procedures are finite dimensional
- controller is infinite dimensional (yet structured & implementable)

---

$^a$Sometimes formulated “as if there were no delay,” yet such approach might lead to disastrous results (Palmor, 1980). More accurate: “with implicit accounting for the delay.”
Smith controller

?

Might not be efficient in disturbance attenuation
(high-gain $\tilde{C} \not\Rightarrow$ high loop gain)

?

Applicable only to open-loop stable plants
(strictly speaking, characteristic polynomial is $\chi_{cl} = D_P(N_P N_C + D_P D_C)$)
Generalizations: DTC with measurable disturbances

Underlying idea\(^a\):

\[ \text{predict the effect of } d \text{ on } y \text{ as well.} \]

\[^a\text{Proposed by Palmor \& Powers (1985)} \]
Watanabe & Ito (1981) proposed to modify the DTC block. Roughly,

$$\Pi = \tilde{P} - Pe^{-sh},$$

$\tilde{P}$ is rational and such that $\Pi$ is stable (always possible). This scheme

😊 applicable to unstable systems.

In the two-stage design the first stage is to be modified as follows:

1. design $\tilde{C}$ for delay-free plant $\tilde{P}$

(instead of $P$).
**Generalizations: observer-predictor (OP)**

- First appears in (Kleinman, 1969) (as the solution of the LQG problem for dead-time systems)

- Rediscovered a decade after that by Manitius & Olbrot (1979) see also (Artstein, 1982) and (Furukawa & Shimemura, 1983)

- Equivalence of MSP and OP shown by Mirkin & Raskin (1999) (a matter of internal change of variables)
Generalizations: ... 

Tens (if not hundreds) of generalizations/modifications:

- modified Smith predictor
- generalized Smith predictor
- improved Smith predictor
- integrated Smith predictor
- unified Smith predictor
- 2DOF Smith predictors
- and even (why not;) fuzzy and neural Smith predictors

The vast majority of these generalizations/modifications, however,

- based solely on ad hoc arguments,

rather than any rigorous analysis.
To compensate or not to compensate?

😊 Seems to be natural and intuitively appealing
  ➔ generic applicability
  ➔ can be traced in biological systems (Miall, Weir, Wolpert, & Stein, 1993)

😊 Exploits the structure of the delay element

😊 Results in infinite-dimensional yet implementable controllers
To compensate or not to compensate?

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😊 Exploits the structure of the delay element

😊 Results in infinite-dimensional yet implementable controllers

😊 “Prediction is very difficult, especially about the future” (N. Bohr)

😊 Considered an ad hoc approach
  ➡ how to compensate (predict)?
  ➡ any rigorous justification?

😊 Is inherently open-loop
  ➡ sensitivity to disturbances?
  ➡ sensitivity to modeling errors?
What are we looking for?

There is clear need for rigorous problem-oriented methods that

- exploit the structure of the delay element

and

- lead to transparent and implementable solutions.
Outline

1. Dead-time compensation: Smith predictor and its generalizations

2. Some delay-oriented techniques:
   - loop shifting
   - factorization & MZ trick
   - extraction
   - ...

3. Controller structures and their interpretations

4. Multiple-delay generalizations (example)

5. Concluding remarks
Preliminaries: \( h \)-truncation

Given \( G = \begin{bmatrix} A & B \\ C & 0 \end{bmatrix} \), its \( h \)-truncation \( \tau_h \{ G \} \) is defined as

\[
\tau_h \{ G \} = G - e^{-sh} \hat{G} = \begin{bmatrix} A & B \\ C & 0 \end{bmatrix} - e^{-sh} \begin{bmatrix} A \\ Ce^{Ah} \\ B \end{bmatrix}
\]

\[= C(I - e^{-(sI-A)h})(sI - A)^{-1}B \quad \text{(entire function)}.
\]

If \( \zeta = \tau_h \{ G \} \omega \), then \( \zeta(t) = C \int_{t-h}^{t} e^{A(t-s)}B\omega(s)ds \) (FIR system).
Preliminaries: $h$-completion

Given $G = \begin{bmatrix} A & B \\ C & 0 \end{bmatrix}$, its $h$-completion $\pi_h\{Ge^{-sh}\}$ is defined as

$$\pi_h\{Ge^{-sh}\} = \tilde{G} - Ge^{-sh} = \begin{bmatrix} A \\ Ce^{-Ah} \end{bmatrix} \begin{bmatrix} B \\ 0 \end{bmatrix} - \begin{bmatrix} A \\ B \end{bmatrix} e^{-sh}$$

$$= e^{-sh}C(e^{(sI-A)h} - I)(sI - A)^{-1}B \quad \text{(entire function)}.$$ 

If $\zeta = \pi_h\{e^{-sh}G\}\omega$, then $\zeta(t) = Ce^{-Ah} \int_{t-h}^{t} e^{A(t-s)}B\omega(s)ds \quad \text{(FIR system)}.$
Loop shifting

The idea is to

- use elementary block-diagram manipulations to get rid of loop delay

✗ idea: Zwart & Bontsema (1997)\(^a\)


\(^a\)Roots in (Curtain & Glover, 1986) and (Curtain & Zhou, 1996), see also (Curtain, Weiss, & Weiss, 1996).
The purpose:

- convert dead-time problem to an equivalent delay-free problem
Loop shifting

The purpose:

→ convert dead-time problem to an equivalent delay-free problem
(Internal) stability is preserved for any $\tilde{P}_{22}$ such that

$$\tilde{P}_{22} - e^{-sh}P_{22} \in H^\infty \text{ (stable)}$$
Loop shifting

\[
\begin{bmatrix}
P_{11} & P_{12} \\
e^{-sh}p_{21} & \bar{p}_{22}
\end{bmatrix}
\]

\(\bar{K}\) is in the

\(\Rightarrow\) modified Smith predictor form
The next step is to pull delay out.
Loop shifting

\[
\begin{bmatrix}
\hat{P}_{11} & P_{12} \\
P_{21} & \tilde{P}_{22}
\end{bmatrix}
\]

\[
e^{sh}P_{11} - \hat{P}_{11}
\]

\[
e^{-sh}\tilde{P}_{22} - e^{-sh}P_{22}
\]

\[
\begin{bmatrix}
P_{22} - e^{-sh}P_{22}
\end{bmatrix}
\]

\[
K
\]

\[
\bar{K} T_{zw}
\]

(Internal) stability is preserved for any \(\hat{P}_{11}\) such that

\[
P_{11} - e^{-sh}\hat{P}_{11} \in \mathcal{H}^\infty \text{ (stable)}
\]
(Internal) stability is preserved for any $\hat{P}_{11}$ such that

\[ P_{11} - e^{-sh}\hat{P}_{11} \in H^\infty \text{ (stable)} \]
Finally, denoting $\Pi_1 = P_{11} - e^{-sh}\hat{P}_{11}$ and $\Pi_2 = \bar{P}_{22} - e^{-sh}P_{22}$ (both stable),

$$T_{zw} = \Pi_1 + e^{-sh}\bar{T}_{zw}$$
Loop shifting: $H^2$ solution

Let’s choose $\Pi_1 = \tau_h\{P_{11}\}$ so that in $\tau_{zw} = \tau_h\{P_{11}\} + e^{-sh}\tilde{T}_{zw}$

- impulse response of $\tau_h\{P_{11}\}$ has support in $[0, h]$
- impulse response of $e^{-sh}\tilde{T}_{zw}$ has support in $[h, \infty)$ for all causal $\tilde{K}$

Thus, for any choice of $\tilde{K}$, $\tau_h\{P_{11}\}$ and $e^{-sh}\tilde{T}_{zw}$ are orthogonal in $H^2$. 

Loop shifting: $H^2$ solution

Therefore:

$$\|T_{zw}\|_2^2 = \|\tau_h\{P_{11}\}\|_2^2 + \|e^{-sh}\tilde{T}_{zw}\|_2^2 = \|\tau_h\{P_{11}\}\|_2^2 + \|\tilde{T}_{zw}\|_2^2$$

and the resulting $K$ is in the DTC (actually, MSP) form.
Loop shifting: SISO $\mathcal{L}^1$ solution

Similarly to the $H^2$ case, $T_{zw} = \tau_h\{P_{11}\} + e^{-sh}\tilde{T}_{zw}$ implies that:

$$\|T_{zw}\|_1 = \|\tau_h\{P_{11}\}\|_1 + \|e^{-sh}\tilde{T}_{zw}\|_1 = \|\tau_h\{P_{11}\}\|_1 + \|\tilde{T}_{zw}\|_1$$

independent of $\tilde{K}$ finite-dimensional

\(^a\)In MIMO case $\|\tau_h\{P_{11}\}\|_1$ can be replaced with a static matrix.
In many robust stability problems $P_{11} = 0 \Rightarrow T_{zw} = e^{-sh}\tilde{T}_{zw}$ and then $\|T_{zw}\|_\infty = \|\tilde{T}_{zw}\|_\infty$. Hence,

$\Rightarrow$ robustness$^a$ of $(P_{22}e^{-sh}, K) \equiv$ robustness of $(\tilde{P}_{22}, \tilde{K})$

(extends (Morari & Zafiriou, 1986) and (Zwart & Bontsema, 1997)).

$^a$To additive and multiplicative, even structured (complex $\mu$) uncertainty.
Loop shifting: conclusions

- rigorous (yet simple)

- exploits the structure of $e^{-sh}$

- naturally produces DTC controllers
  (actually, proves that the modified Smith predictor is both $H^2$ and $L^1$ "optimal")
MZ trick

The purpose is to

- extract delay terms from J-spectral factorization


Preliminary: \( J \)-spectral factorization

Given self-conjugate and bi-proper

\[
\Phi = \begin{bmatrix}
\Phi_{11} & \Phi_{12} \\
\Phi_{21} & \Phi_{22}
\end{bmatrix} = \Phi^\sim,
\]

find bi-proper, outer, etc

\[
\Theta = \begin{bmatrix}
\Theta_{11} & \Theta_{12} \\
\Theta_{21} & \Theta_{22}
\end{bmatrix}
\]

such that

\[
\Phi^{-1} = \Theta J \Theta^\sim,
\]

where \( J = \begin{bmatrix} 1 & -1 \end{bmatrix} \) is the signature matrix.

All admissible \( \mathcal{H}^\infty \) controllers are then given by

\[
C_r(\Theta, Q) \triangleq (\Theta_{11} Q + \Theta_{12})(\Theta_{21} Q + \Theta_{22})^{-1} \quad \text{for any } \|Q\|_\infty < 1.
\]
MZ trick (contd)

In the dead-time case

\[ \Phi = \begin{bmatrix} \Phi_{11} & e^{sh}\Phi_{12} \\ e^{-sh}\Phi_{21} & \Phi_{22} \end{bmatrix} \] (infinite dimensional).
MZ trick (contd)

In the dead-time case

\[
\Phi = \begin{bmatrix}
\Phi_{11} & e^{sh}\Phi_{12} \\
e^{-sh}\Phi_{21} & \Phi_{22}
\end{bmatrix}
\text{ (infinite dimensional)}.\]

Schur decomposition of \( \Phi \) (\( \Phi_{22} \) is invertible):

\[
\Phi = \begin{bmatrix}
I & e^{sh}\Phi_{12}\Phi_{22}^{-1} \\
0 & I
\end{bmatrix}
\begin{bmatrix}
\Phi_{11} - \Phi_{12}\Phi_{22}^{-1}\Phi_{21} & 0 \\
0 & \Phi_{22}
\end{bmatrix}
\begin{bmatrix}
I & 0 \\
e^{-sh}\Phi_{22}^{-1}\Phi_{21} & I
\end{bmatrix}
\]

On the other hand,

\[
\begin{bmatrix}
I \\
e^{-sh}\Phi_{22}^{-1}\Phi_{21}(\mp\Phi_{22}^{-1}\tilde{\Phi}_{21}) & I
\end{bmatrix}
= \begin{bmatrix}
I & 0 \\
\Phi_{22}^{-1}\tilde{\Phi}_{21} & I
\end{bmatrix}
\begin{bmatrix}
I & 0 \\
\Pi & I
\end{bmatrix},
\]

where \( \Pi = -\pi h\{e^{-sh}\Phi_{22}^{-1}\Phi_{21}\} = e^{-sh}\Phi_{22}^{-1}\Phi_{21} - \Phi_{22}^{-1}\tilde{\Phi}_{21} \) is FIR (\( \Rightarrow \) stable).
MZ trick (contd)

Thus,

\[
\Phi = \begin{bmatrix} I & \Pi^- \\ 0 & I \end{bmatrix} \left[ \begin{array}{cc} \Phi_{11} - \Phi_{12} \Phi_{22}^{-1} \Phi_{21} + \tilde{\Phi}_{12} \Phi_{22}^{-1} \tilde{\Phi}_{21} & \tilde{\Phi}_{12} \\ \tilde{\Phi}_{21} & \Phi_{22} \end{array} \right] \begin{bmatrix} I & 0 \\ \Pi & I \end{bmatrix}
\]

and

\[
\Theta = \begin{bmatrix} I & 0 \\ -\Pi & I \end{bmatrix} \tilde{\Theta},
\]

where \( \tilde{\Theta} \) is (finite-dimensional) J-spectral factor of

\[
\tilde{\Phi} = \begin{bmatrix} \Phi_{11} - \Phi_{12} \Phi_{22}^{-1} \Phi_{21} + \tilde{\Phi}_{12} \Phi_{22}^{-1} \tilde{\Phi}_{21} & \tilde{\Phi}_{12} \\ \tilde{\Phi}_{21} & \Phi_{22} \end{bmatrix}.
\]
The set of all admissible $H^\infty$ controllers is

$$K = (\Theta_{11}Q + \Theta_{12})(\Theta_{21}Q + \Theta_{22})^{-1} \quad \text{for any } \|Q\|_\infty < 1.$$ 

or, equivalently,
Remember that $\Theta = \begin{bmatrix} I & 0 \\ -\Pi & I \end{bmatrix} \tilde{\Theta}$. Hence:
MZ trick: controller structure

Note:

\[
\begin{bmatrix}
  u \\
  y
\end{bmatrix} = \begin{bmatrix}
  I & 0 \\
  -\Pi & I
\end{bmatrix} \begin{bmatrix}
  \zeta \\
  \eta
\end{bmatrix} \iff \begin{cases}
  u = \zeta \\
  \eta = y + \Pi \zeta
\end{cases}
\]
MZ trick: controller structure

\[
\begin{bmatrix}
  u \\
  y
\end{bmatrix}
=\begin{bmatrix}
  I & 0 \\
  -\Pi & I
\end{bmatrix}
\begin{bmatrix}
  \zeta \\
  \eta
\end{bmatrix}
\iff
\begin{cases}
  u = \zeta \\
  \eta = y + \Pi \zeta
\end{cases}
\]
MZ trick: controller structure

where

- $\tilde{\Theta}$ is finite dimensional;
- $Q$ is free parameter satisfying $\|Q\|_\infty < 1$;
- $\Pi$ is infinite-dimensional FIR system of the form $\tilde{p}_a - e^{-sh}p_a$. 
MZ trick: controller structure

where

- $\tilde{\Theta}$ is finite dimensional;
- $Q$ is free parameter satisfying $\|Q\|_{\infty} < 1$;
- $\Pi$ is infinite-dimensional FIR system of the form $\tilde{p}_a - e^{-sh}p_a$.

This is DTC!
MZ trick: conclusions

- rigorous

- exploits the structure of $e^{-sh}$

- naturally produces DTC controllers
  (results in the so-called $H^\infty$ DTC)
The idea is to treat $e^{-sh}$ as (causality) constraint imposed on the controller.


Roots in (Mirkin & Rotstein, 1997), where applied in the context of sampled-data systems.
Extraction: driving idea

Conventional treatment: $e^{-sh}$ is part of the plant. This yields

- unconstrained controller $K$ for infinite-dimensional plant $P_h$
Extraction: driving idea

Conventional treatment: $e^{-sh}$ is part of the plant. This yields

- unconstrained controller $K$ for infinite-dimensional plant $P_h$

Proposed treatment: $e^{-sh}$ is part of the controller. This yields

- constrained controller $K_h$ for finite-dimensional plant $P$
Extraction: constraints

\[ k(t, s), \text{ causal system with DT} \]

\[ k(t, s), \text{ causal system} \]
Extraction: constraints

\[ k(t, s), \text{ causal system with DT} \]

\[ k(t, s), \text{ causal system} \]

DT controllers can be extracted from delay-free parametrizations
Extraction (contd)

4-block dead-time problem

\[ z \overrightarrow{\rightarrow} w \]
\[ y \overrightarrow{\rightarrow} u \]
\[ e^{\sigma T} \]
\[ K \]
\[ T_{zw} \]

\[ K \]
\[ T_{zw} \]

4-block delay-free problem

1-block dead-time problem
Extraction (contd)

4-block delay-free problem:

😊 well understood

😊 finite-dimensional solvability conditions

1-block dead-time problem:

😊 reduces to a finite-dimensional open-loop problem over \([0, h]\)

😊 shows “the cost of delay”
Extraction: conclusions

- rigorous (yet simple)

- exploits the structure of $e^{-sh}$

- produces DTC controllers
  (results in the so-called $H^\infty$ DTC)
Some other techniques

✗ Lifting-based approach (Nagpal & Ravi, 1994, 1997)
  ➡ Elegant solvability conditions for the $H^\infty$ problem are derived
  ➡ Results in “periodic” solution

✗ Game-theoretic methods (Tadmor, 1995, 2000)
  ➡ The game partitioned into finite-horizon indefinite single player optimization and an infinite-horizon delay-free $H^\infty$ game.
  ➡ Matrix-based solution provides explicit formulae for the solution of an operator Riccati equation for an equivalent abstract model
  ➡ Results in observer-predictor form of the controller

✗ “Peeling-off” the abstract solution (Kojima & Ishijima, 1996, 2001)
  ➡ The abstract solution is split into a matrix equation and a closed-form distributed-delay operator.
  ➡ Results in observer-predictor form of the controller
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Here

\[ \Pi_2 = \pi_h \{ P_{22} e^{-sh} \} = \tilde{P}_{22} - P_{22} e^{-sh} \]

and the trick is to

- pull delay out of the internal loop \( u \odot y_c \): \( y_c = \tilde{P}_{22} u + v \)

so that \( \tilde{K} \) is the \( H^2 \)-optimal controller for (finite-dimensional) \( \tilde{P}_{22} \).
Here

\[ \Pi_\infty = \pi_h \{ \mathcal{F}_u (P, \frac{1}{\gamma^2} \rho_1) e^{-sh} \} = \bar{\rho}_\gamma - P \gamma e^{-sh}, \]

where \( P \gamma = \mathcal{F}_u (P, \frac{1}{\gamma^2} \rho_1) = P_{22} + P_{21} (\gamma^2 I - \bar{\rho}_{11} \rho_{11})^{-1} \bar{\rho}_{11} \rho_{12}. \) Yet now \( u \odot y_c \) still contains delay:

\[ y_c = \bar{\rho}_\gamma u + v - e^{-sh} P_{21} (\gamma^2 I - \bar{\rho}_{11} \rho_{11})^{-1} \bar{\rho}_{11} \rho_{12} u. \]
$H^\infty$ DTC: rationale

Delay in $u \circlearrowleft y_c$ would be compensated if

$$v = e^{-sh}P_{21}(\gamma^2 I - \bar{P}_{11}^\sim P_{11})^{-1}\bar{P}_{11}^\sim P_{12} u$$
Delay in $u \circ y_c$ would be compensated if

$$v = e^{-sh}p_{21}(\gamma^2 I - p_{11}^- p_{11})^{-1}p_{11}^- p_{12} u$$

or, equivalently,

$$w = (\gamma^2 I - p_{11}^- p_{11})^{-1}p_{11}^- p_{12} u$$
\[ \text{H}^{\infty} \text{ DTC: rationale} \]

where \( P_w \doteq (\gamma^2 I - P_{11}^\sim P_{11})^{-1}P_{11}^\sim P_{12} \) and

\[ \Pi_w \doteq \pi_h \{ e^{-sh}P_{21}(\gamma^2 I - P_{11}^\sim P_{11})^{-1}P_{11}^\sim P_{12} \}. \]

Thus, \( \text{H}^{\infty} \) DTC can be thought of as the

\( \Rightarrow \) \( \text{H}^2 \) DTC with measurable disturbance

when \( w = \tilde{w} \doteq (\gamma^2 I - P_{11}^\sim P_{11})^{-1}P_{11}^\sim P_{12} u \).
Where this \( \tilde{w} = (\gamma^2 I - P_{11}^\sim P_{11})^{-1}P_{11}^\sim P_{12} u \) comes from?

Let \( \begin{bmatrix} P_{21} & P_{22} \end{bmatrix} \) be described as

\[
\begin{align*}
\dot{x} &= Ax + B_1 w + B_2 u \\
y &= C_2 x + D_{21} w
\end{align*}
\]

Then \( \tilde{w} \) is generated by

\[
\begin{align*}
-\dot{\lambda} &= A' \lambda + C'_1 z \\
\tilde{w} &= \frac{1}{\gamma^2} B'_1 \lambda
\end{align*}
\]

(where \( z = C_1 x + D_{12} u \) is regulated output)

Thus, \( \tilde{w} \) is the worst case disturbance for the open-loop \( H^\infty \) problem.
$H^\infty$ DTC: rationale (contd)

$H^\infty$ DTC would be the

$\Rightarrow$ $H^2$ DTC with measurable disturbance

if $w$ were the

$\Rightarrow$ worst case disturbance for the open-loop $H^\infty$ problem.
$H^\infty$ DTC: conclusions

Thus,

$\Rightarrow H^\infty$ dead-time compensator “predicts” both $y$ and $w$

If $w$ were available $h$ time units ahead, DTC should account for it, see (Palmor & Powers, 1985). Yet since $w$ is unavailable, $\Pi^\infty$ attempts to predict $w$ and it does this using the \textit{worst-case open-loop} scenario.
Thus,

- $H^\infty$ dead-time compensator “predicts” both $y$ and $w$

  If $w$ were available $h$ time units ahead, DTC should account for it, see (Palmor & Powers, 1985). Yet since $w$ is unavailable, $\Pi_\infty$ attempts to predict $w$ and it does this using the worst-case open-loop scenario.

“The best way to predict the future is to invent it”

Alan Kay
Outline

① Dead-time compensation: Smith predictor and its generalizations

② Some delay-oriented techniques:
  ➡️ loop shifting
  ➡️ factorization & MZ trick
  ➡️ extraction
  ➡️ …

③ Controller structures and their interpretations

④ Multiple-delay generalizations (example)

⑤ Concluding remarks
The problem is to

- reconstruct $u$ from noisy delayed measurements

$$y_i(t) = u(t - h_i) + \sigma_i v_i(t - h_i).$$

We suppose that

- $u$ is a bounded-velocity signal
- $h_2 = h_1 + h_\delta$, $h_\delta \geq 0$
- performance is measured by the $H^\infty$ norm of the error
The idea is to

- split the problem to a nested sequence of *adobe problems*

which are simpler problems with a *single delay* in a *part of the channels*.

When combined,

- *adobe solutions* fall into place

leading to subsequent simplifications and closed form solution.
Multiple-delay example: results for $\gamma = \sigma_1$

This performance level corresponds to the case when

- only one delay-free sensor is used

$(h_1 = h_2 = 0$ and $\sigma_2 \to \infty$).

Thus, we attempt to recover the optimal delay-free performance by the use of additional sensor.

1. The first adobe problem (with $\Lambda = \begin{bmatrix} 1 & 0 \\ 0 & e^{-h_\delta s} \end{bmatrix}$) is solvable $\forall h_\delta$

2. The second adobe problem (with $\Lambda = e^{-h_1 s}I_2$) is solvable for

$$h_1 < \bar{h}_1 = \sigma_1 \arctan \frac{\sigma_1}{\sigma_2 + h_\delta} \leq \frac{\pi}{2} \sigma_1.$$

Note that

- the effect of delay $\equiv$ the effect of noise

in the second measurement channel.
Multiple-delay example: resulting estimator

\[ \tilde{K} = \frac{1}{\sqrt{\sigma_1^2 + (h_\delta + \sigma_2)^2}} \left[ h_\delta + \sigma_2 \quad \frac{\sigma_1^2}{\sigma_2} \right] \]

where \( \tilde{K} \) is

and \( \Pi_\bullet \) are FIR DTC blocks.
Multiple-delay example: resulting estimator

\[ K \]

\[ \tilde{K} \]

\[ u \]

\[ \Pi_{\delta_1} \]

\[ \Pi_{\delta_2} + e^{-h_1s}\Pi_b \]

\[ e^{-h_1s} \]

\[ e^{-h_2s} \]

\[ \Pi_f \]

\[ y \]

✗ \( \Pi_{\delta_i} \) and \( \Pi_b \) are counterparts of the single-delay DTC

✗ \( \Pi_f \) (interchannel interconnection) has no single-delay counterpart

There is still appears to be no satisfactory generalization of the Smith predictor to the multiple delay case. Optimization-based solution may suggest one.
Outline

① Dead-time compensation: Smith predictor and its generalizations

② Some delay-oriented techniques:
   ➡ loop shifting
   ➡ factorization & MZ trick
   ➡ extraction
   ➡ ...

③ Controller structures and their interpretations

④ Multiple-delay generalizations (example)

⑤ Concluding remarks
Concluding remarks

✗ Analysis of DT systems benefits from problem-oriented treatment
   there is nice structure deserved to be fully exploited

✗ DTC structure does have rigorous justifications
   and is not “yet another ad hoc approach”
Some open problems

- Rationale behind multiple delay DTC
- Embedding of DTC into LMI & $C^o$ frameworks
- Sensitivity to uncertainty in delay
- Time-varying delays
- Not-in-the-loop delays
- Extensions to different infinite-dimensional problems
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