Distributed feedforward control of wind farms: prospects and open problems

Maxim Kristalny

Department of Automatic Control, LTH

Joint work with Daria Madjidjian and Anders Rantzer

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Preliminaries

Wind power constitutes

- 2% of power production over the world
- more than 20% of power production in Denmark

Demand in wind power continues to grow
Preliminaries

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Demand in wind power continues to grow

Growth of turbines dimensions

- complex systems producing up to 7.5 MW
- structural loads become a central issue

by L. Y. Pao and K. E. Johnson, ACC 2009
Preliminaries

Wind power constitutes
- 2% of power production over the world
- more than 20% of power production in Denmark

Demand in wind power continues to grow

Construction of large scale wind farms (economically beneficial)

- today each turbine in a farm is controlled separately

Are there better options?
Distribution of power between turbines

Wake effects - upwind turbine partially block wind flow
Nowadays, each turbine tries to extract the maximum
Is this an optimal way?

- Decrease in upwind turbine power may increase overall power

(D. Madjidian and A. Rantzer, 2011)
Load reduction in turbines

Load control is essential in modern wind turbines

Load reduction contributes to cost efficiency of turbines
Load reduction in turbines located in farms

The main idea:
Accounting for and communicating with neighbours might be beneficial

- sharing wind speed measurements
- cooperation in terms of power production
Load reduction in turbines located in farms

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Accounting for and communicating with neighbours might be beneficial

- sharing wind speed measurements
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Wind farm models contain delays due to wind propagation

Distributed control is required due to large scale and modularity demand
Outline

1 Preview control of individual turbine
   - individual turbine model
   - problem solution
   - simulation results

2 Cooperative control of entire farm
   - wind farm model
   - quadratic invariance
   - solution
   - preliminary simulation results
Outline

1. Preview control of individual turbine
2. Cooperative control of entire farm
Turbine model

Produced power: \[ P = T_g \cdot \omega_g \]
Control of individual turbine

Turbine model

Produced power: \( P = T_g \cdot \omega_g \)

Internal controller adjusts \( \beta \) and \( T_g \) to maintain operating point.
Standard internal controller

Several logical switches and PIDs
Maintains rotor speed according to:

Three operating regions:

I  Wind speed too low to produce power  (the rotor is frozen)
II  Energy available from wind is less than required  ($\beta = 0$, $\omega$ depends on wind)
III  Energy available from wind is more than required  (rated $\omega$, $\beta$ depends on wind)
Standard internal controller

Several logical switches and PIDs
Maintains rotor speed according to:

Three operating regions:

I Wind speed too low to produce power  (the rotor is frozen)
II Energy available from wind is less then required  \( \beta = 0, \omega \) depends on wind
III Energy available from wind is more then required  (rated \( \omega \), \( \beta \) depends on wind)

Assume operation in region III
Keeping standard internal controller is restrictive

- prevents direct access to pitch and generator torque
- leaves power reference as the only control signal

At the same time, this

- simplifies the problem
- facilitates experiments in existing wind farms
Standard internal controller

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Problems formulations discussed below

can be extended for the case without internal controller
Turbine with internal controller

NREL 5 MW turbine with standard internal controller (in operating region III)

Inputs:
\[ V \text{ - wind speed}; \; p_{\text{ref}} \text{ - power reference}; \]

Outputs:
\[ F \text{ - thrust force}; \; T_g \text{ - generator torque} \]
\[ \omega \text{ - rotor speed}; \; \beta \text{ - pitch angle}; \]

Model neglects electrical circuit dynamics (power production equals power reference)

Model is linearized around operating point (all signals represent deviations from nominal values)
Problem formulation

Measured disturbance attenuation

Deviations of wind speed $V = \text{disturbances measured with noise } n$

The aim is to keep deviations of turbine outputs small
Problem formulation

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Availability of preview is captured by delay operator $e^{-sh}$
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Weights

Thrust force - accounts for tower oscillations

\[ W_F = k_F \frac{s + \omega_{\text{twr}}}{s^2 + \omega_{\text{twr}}^2}. \]

Generator torque - accounts for drive train oscillations

\[ W_T = k_T \frac{s + \omega_{\text{shf}}}{s^2 + \omega_{\text{sft}}^2}. \]

Control signal - no change of set point for permanent wind changes
(pre-power reference)

\[ W_u = k_p \frac{(0.1s + 1)}{s} \cdot \frac{(s + \omega_{\text{twr}})^2}{s^2 + 0.02 \omega_{\text{twr}} + \omega_{\text{twr}}^2}. \]

The rest of the weights are static.
Model matching

The problem can be cast as model matching - unified framework for estimation and feedforward control

\[
G_1 = \begin{bmatrix} W_z P_v W_v & 0 \\ 0 & 0 \end{bmatrix} \quad G_2 = \begin{bmatrix} W_v & W_n \end{bmatrix} \quad G_3 = \begin{bmatrix} -W_z P_u \\ -W_z \end{bmatrix}
\]
Model matching

Problem statement

Given LTI $G_1$, $G_2$, $G_3$ and $h > 0$, find stable and causal $K$ such that

- $T$ is input/output stable (asymptotic performance)
- $\|T\|_2$ is minimal (transient performance)
Model matching

One-side problem with either $G_2 = I$ or $G_3 = I$
- corresponds to problem without measurement noise
- solved in both $H^2$ and $H^\infty$ settings
  
  (Tomizuka, 1975; Moelia and Meinsma, 2006; Mirkin and Tadmor, 2007)

Extension to general two-side problem is not trivial
(due to combination of asymptotic and transient performance requirements)
- without stability constraints the extension could be straightforward
- with stability constraints solved,
  yet is not readily extendable for the case with preview.
  
  (Liu, Zhang and Mita, 1997)
Model-matching vs standard four-block stabilization

Model matching stabilization

\[ G_3 \quad K \quad G_2 \]

Standard four-block stabilization

\[ G_{11} \quad G_{12} \quad G_{21} \quad G_{22} \]

- Unstable modes ⇔ physical instabilities
- Internal stability requirement
- Unstable cancelations prohibited
Model-matching vs standard four-block stabilization

Model matching stabilization

- Unstable modes ⇔ external signal dynamics
- No internal stability requirement
- Unstable cancelations eligible

Standard four-block stabilization

- Unstable modes ⇔ physical instabilities
- Internal stability requirement
- Unstable cancelations prohibited

Not a special case of the standard problem
Stabilization

Two-side stabilization is more complicated than one-side
- related to bilateral Diophantine equations
- state-space solution involves Sylvester equations
- cumbersome parameterization of stabilizing solutions
Stabilization

Two-side stabilization is more complicated than one-side
- related to bilateral Diophantine equations
- state-space solution involves Sylvester equations
- cumbersome parameterization of stabilizing solutions

Assumptions

\( \mathcal{A}_1 \) Unstable poles of \( G_2 \) do not coincide with zeros of \( G_3 \)

\( \mathcal{A}_2 \) Unstable poles of \( G_3 \) do not coincide with zeros of \( G_2 \)

satisfied in practical problems and lead to convenient parameterization

\[
K = K_p + \tilde{M}_3 Q M_2, \quad Q \in H^\infty
\]

Stability constraints released without changing problem structure

\[
T = \left( G_1 - G_3 K_p G_2 \right) + \tilde{N}_3 Q \tilde{N}_2
\]
Optimization

\[ \min_{Q \in H^2} \left\| e^{-sh} \bar{G}_1 - \bar{G}_3 Q \bar{G}_2 \right\|_2 \]

Following the steps of the one-side solution:

- reduction to one-block problem (square completion)
- one-block problem solution (projection theorem)
- handling preview element (completion operator)

Nontrivial steps required to derive state-space formulae for two-side case
Optimization

Modified Riccati equations

\[(A+B_2 F_t)'X + X(A+B_2 F_t) - (XB_2 + C'D_3')(B_2'X + D_3 C) + C' C = 0,\]
\[(A+L_tC_2)Y + Y(A+L_tC_2)' - (YC_2 + BD_2')(C_2 Y + D_2 B') + BB' = 0\]

- standard $H^2$ AREs with shifted A-matrices
- $F_t$ and $L_t$ defined by Sylvester equation (from stabilization solution)
- solvable for any stabilizable problem
- solution satisfies standard Riccati (but is not stabilizing for standard Riccati)
Optimization

Modified Riccati equations

\[(A+B_2F_t)'X + X(A+B_2F_t) - (XB_2 + C'D_3')(B'_2X + D_3C) + C'C = 0,\]
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- solvable for any stabilizable problem
- solution satisfies standard Riccati (but is not stabilizing for standard Riccati)

Alternative to the notion of semi-stabilizing ARE solutions proposed in Liu, Zhang and Mita, 1997
Solution

Based on:

Two Sylvester equations (stabilization)

Two Riccati equations (optimization)

- not affected by preview length
- shifted by terms from Sylvester
**Solution**

Based on:

Two Sylvester equations  
(stabilization)

Two Riccati equations  
(optimization)

- not affected by preview length
- shifted by terms from Sylvester

Optimal solution structure:

\[
M_{1/2/3} \quad \text{finite dimensional} \\
M_1 \quad \text{optimal for } h = 0
\]
Solution

Based on:

Two Sylvester equations

Two Riccati equations
  - not affected by preview length
  - shifted by terms from Sylvester

Optimal solution structure:

\[ M_{1/2/3} \text{ finite dimensional } h \text{- independent} \]

\[ M_1 \text{ optimal for } h = 0 \]
Performance vs. preview length

- The relevant scale of preview is a number of seconds
- 98% of improvement achieved with 1.7 sec preview
Simulation results

Control signal

(change of power production)
Outline

1. Preview control of individual turbine
2. Cooperative control of entire farm
Wind farm model

Wake effects: mean speed deficit, increase of turbulence
- important in quasi-static analysis of farm
  (distribution of nominal powers among turbines in farm)
- less important for dynamics around specified operating point

Neglect influence of pitch on wind flow

Wind propagation modeled as delay and additive noise
Distributed feedforward control

Preview control from previous part can be applied to each turbine in farm

Drawback:
adjustment of turbines power $\Rightarrow$ fluctuations in overall power production

Cooperation between turbines may be beneficial
- requires formulation that takes the entire farm into account
Distributed feedforward control

We keeping this control scheme, but optimize for the entire farm

Inputs: $\bar{w} := \begin{bmatrix} V & v_1 & \cdots & v_N & n_1 & \cdots & n_N \end{bmatrix}'$

Outputs: $\bar{e} := \begin{bmatrix} z_1 & \cdots & z_N & u_1 & \cdots & u_N & \sum u_i \end{bmatrix}'$

The problem can be cast as model matching
Decentralized model matching

\[
\Lambda := \text{diag}\{1, e^{-sh}, e^{-2sh}, \ldots\}
\]

\[
K = \text{diag}\{K_1 \ldots K_N\} \in H^\infty
\]
Decentralized model matching

\[ \Lambda := \text{diag}\{1, e^{-sh}, e^{-2sh}, \ldots\} \]

\[ K = \text{diag}\{K_1, \ldots, K_N\} \in H^\infty \]

More complicated than in previous part:

- Parameter \( K \) constrained to be diagonal
  - Communication only with closest upwind neighbors
  - Various communication patterns can be considered
    (various structural constraints on \( K \))

- Complicated structure of delays
  - Uniform delay \( e^{-sh} \) corresponds to availability of preview
  - Multichannel delay \( \Lambda \) is due to wind propagation
Decentralized model matching

\[ \Lambda := \text{diag}\{1, e^{-sh}, e^{-2sh}, \ldots\} \]

\[ K = \text{diag}\{K_1 \ldots K_N\} \in H^\infty \]

Open problems:

- Decentralized model matching stabilizing
  - May stability constraints be released without changing problem structure?

- Decentralized model matching optimization
  - Relevant in various distributed control problems

- How to handle delays?
  - Discretization leads to numerical difficulties
  - Does there exist a convenient solution like in centralized case?
Decentralized model matching

\[ \Lambda := \text{diag}\{1, e^{-sh}, e^{-2sh}, \ldots\} \]

\[ K = \text{diag}\{K_1, \ldots, K_N\} \in H^\infty \]

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Distributed control

- Constraint on structure of controller (plant may also have structure)
- No analytical solutions available in the literature
- Generally, optimal controller may not be linear

(H. S. Witsenhausen, 1968)
Distributed control

Hope to find tractable solutions for some special cases:

- positive systems  
  (Tanaka and Langbord, 2010; Rantzer, 2011)
- quadratically invariant problems  
  (Rotkowitz and Lall, 2006)
Quadratically invariant problems

- Relation between structures of controller and $G_{22}$
- This is a “small” class of distributed control problems
- Yet, it has practical motivation
Formation control

Goal: Follow trajectory, while keeping formation

- $u_i$ - individual control signal
- $x_i$ - absolute position

- Each agent has individual control input
- Absolute position of each agent is measured separately

- $G_{22}$ has diagonal structure
Formation control

Quadratically invariant communication patterns
(for diagonal $G_{22}$)

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No communication
(each agent measures only its own position)

Hierarchical communication
(each agent measures himself and all predecessors)

Leader based alignment
(each agent measures himself and the leader)
Other examples

Wind farm without internal controllers is quadratically invariant
(feedbacks based on local measurements)

More generally, problems, in which agents
- have independent dynamics
- are coupled with common disturb. and control objectives
are quadratically invariant
Quadratic invariance

Youla parameterization can be applied
- reduces the problem to model matching
- constraint on Youla parameter structure
  (the same constraint as on original controller)

Arbitrary structure can be transformed into diagonal

(Rotkowitz, 2010)
Decentralized model matching

Any quad. inv. problem can be reduced to decentralized model matching

Analytical solution is missing:
- Is it possible to derive closed form formulae for optimal solution?
- What is the structure of optimal solution?
- What is the order of optimal solution?

So far, only three agents special case with triangular structure is solved

(Swigart and Lall, 2010)
Notation

Kronecker product
\[
\begin{bmatrix}
  a_{11} & a_{12} \\
  a_{21} & a_{22}
\end{bmatrix}
\otimes
\begin{bmatrix}
  b_{11} & b_{12} \\
  b_{21} & b_{22}
\end{bmatrix}
=\begin{bmatrix}
  a_{11}b_{11} & a_{11}b_{12} & a_{12}b_{11} & a_{12}b_{12} \\
  a_{11}b_{21} & a_{11}b_{22} & a_{12}b_{21} & a_{12}b_{22} \\
  a_{21}b_{11} & a_{21}b_{12} & a_{22}b_{11} & a_{22}b_{12} \\
  a_{21}b_{21} & a_{21}b_{22} & a_{22}b_{21} & a_{22}b_{22}
\end{bmatrix}
\]

Khatri-Rao product
\[
\begin{bmatrix}
  a_{11} & a_{12} \\
  a_{21} & a_{22}
\end{bmatrix}
\odot
\begin{bmatrix}
  b_{11} & b_{12} \\
  b_{21} & b_{22}
\end{bmatrix}
=\begin{bmatrix}
  a_{11}b_{11} & a_{12}b_{12} \\
  a_{11}b_{21} & a_{12}b_{22} \\
  a_{21}b_{11} & a_{22}b_{12} \\
  a_{21}b_{21} & a_{22}b_{22}
\end{bmatrix}
\]

Hadamard product
\[
(\text{element wise})
\begin{bmatrix}
  a_{11} & a_{12} \\
  a_{21} & a_{22}
\end{bmatrix}
\circ
\begin{bmatrix}
  b_{11} & b_{12} \\
  b_{21} & b_{22}
\end{bmatrix}
=\begin{bmatrix}
  a_{11}b_{11} & a_{12}b_{12} \\
  a_{21}b_{21} & a_{22}b_{22}
\end{bmatrix}
\]

Operators vec and dvec
\[
\text{vec} \left( \begin{bmatrix}
  a_{11} & a_{12} \\
  a_{21} & a_{22}
\end{bmatrix} \right) = \begin{bmatrix}
  a_{11} \\
  a_{21} \\
  a_{12} \\
  a_{22}
\end{bmatrix}, \quad
\text{dvec} \left( \begin{bmatrix}
  a_{11} & 0 \\
  0 & a_{22}
\end{bmatrix} \right) = \begin{bmatrix}
  a_{11} \\
  a_{22}
\end{bmatrix}
\]

Idea to apply Kronecker product to decentralized model matching by K. Park, 2008
Reduction to one-side problem

Applying vec operator

\[ \text{vec}(T) = \text{vec}(G_1) - \text{vec}(G_3KG_2) \]

Using properties of Kronecker product

\[ \text{vec}(T) = \text{vec}(G_1) - (G'_2 \otimes G_3)\text{vec}(K) \]

Removing redundant columns

\[ \text{vec}(T) = \text{vec}(G_1) - (G'_2 \odot G_3)d\text{vec}(K) \]

Problem reduced to one-side model matching without structural constraints
The number of columns does not grow
Decentralized model matching and Hadamard product

Solution in terms of spectral factorization

\[ U \sim U = (G'_2 \odot G_3) \sim (G'_2 \odot G_3) \]

Original problem dimensions are preserved

Using properties of Khatri-Rao product

\[ U \sim U = (G_2 G'_2) \sim (G_3 G'_3) \]

Spectral factorization of Hadamard product of matrices associated with centralized solution

Explicit state-space formulae for Hadamard product are needed
Structure of delays

Neglect $v_i$, $n_i$ and assume $N = 2$

Can it be solved via splitting the time axes?

(similarly to A. Moelja and G. Meinsma, 2006; G. Marro and E. Zattoni, 2005; G. Tadmor, 1997)
Splitting the time axis

\[ T = e^{-sh}G_{11} + e^{-2sh}G_{12} - G_{21}K_1 - e^{-sh}G_{22}K_2 \]
Splitting the time axis

\[ T = e^{-sh}G_{11} + e^{-2sh}G_{12} - G_{21}K_1 - e^{-sh}G_{22}K_2 \]
Approximate solution

\[ \Lambda = \text{diag}\{1, e^{-sh}, e^{-2sh}, \ldots \} \]

\[ K = \text{diag}\{K_1 \ldots K_N\} \in H^\infty \]

Open problems:
- Decentralized model matching stabilization
  Shifting imaginary axis poles to OLHP

- Decentralized model matching optimization
  Frequency domain solution in terms of Hadamard product

- How to handle delays?
  Discretization of time axis (feasible for small number of turbines)
Optimal controllers

Impulse responses of optimal controllers

Each controller takes care of downwind turbines

(peaks in time multiples of $h$)
Simulation results

- Slight decrease in tower oscillations
- Demands more deviations in individual turbine powers
Simulation results - improvement in overall power production

- decrease in deviations of overall farm power production
- without deterioration in terms of load reduction
Summary

- Feedforward control based on previewed wind speed measurements
  - individual turbine control
  - distributed control of entire farm

- Cooperation and use of preview are advantageous

- The results motivate more detailed study
  - without internal controllers
  - other communication patterns
  - realistic model of wind propagation
Summary

- Feedforward control based on previewed wind speed measurements
  - individual turbine control
  - distributed control of entire farm

- Cooperation and use of preview are advantageous

- The results motivate more detailed study
  - without internal controllers
  - other communication patterns
  - realistic model of wind propagation

Open theoretical problems:
Decentralized model matching with complicated structure of delays

- stabilization
- optimization (solution based on Hadamard product)
Thank you for attention!