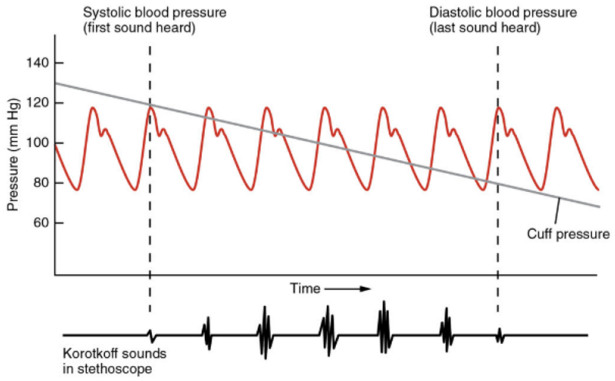
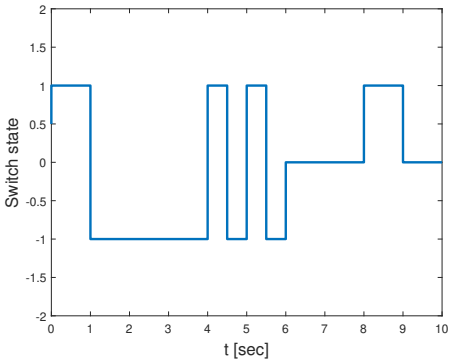
# Linear Systems (034032)

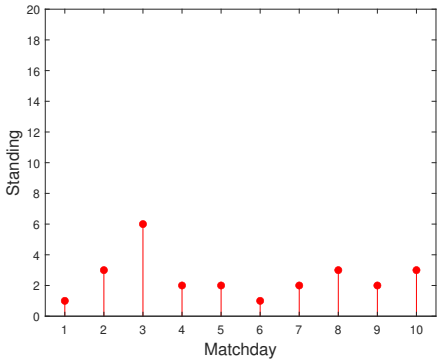
TUTORIAL 1



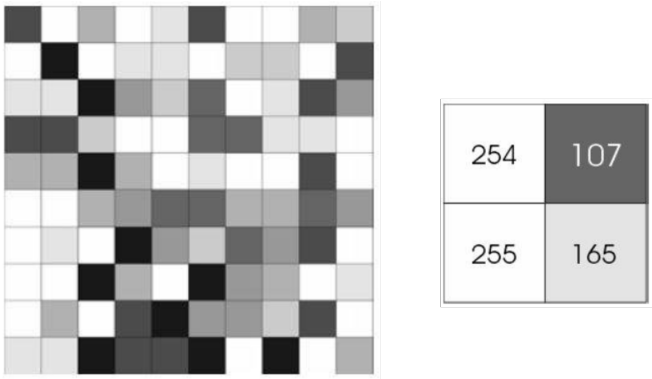

(a) Blood pressure cycle




(b) A 3-state switch value




(c) Liverpool FC weekly Premier League standings in 10 first matchdays of season 2021-2022




(d) A 8 bit grayscale image $10 \times 10$ pixel

Fig. 1: Signals for Question 1

**Question 1.** Describe properties (domains and codomains) of the signals in Fig. 1.

**Question 2.** Consider a mass rotating inside a cylinder depicted in Fig. 2. The mass, whose moment of inertia is $J$, is attached to a torsion spring, whose torsion coefficient is $k_T$. An external torque $\tau$ acts on the mass and friction between the mass and the cylinder is assumed to generate a viscous friction torque $\tau_c = -c_T\dot{\theta}$. Find the relation between input signal $\tau$ and output signal $\theta$.

**Question 3.** Consider the following system described in Fig. 3.

The input is a force $f$ acting on the cart $m_1$, which is constrained to slide without friction in the horizontal direction. A pendulum, mass $m_2$, length $l$, is attached to the cart and is free to rotate around its axis. The outputs are the position of the cart $x$ and the angle of the pendulum $\theta$. Write the equations of motion.

## Norms

Norms are a class of functions that enable us to quantify the size of a vector by assigning a nonnegative

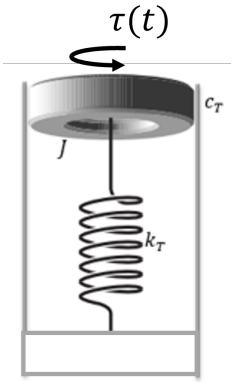



Fig. 2: Spring mass damper system.

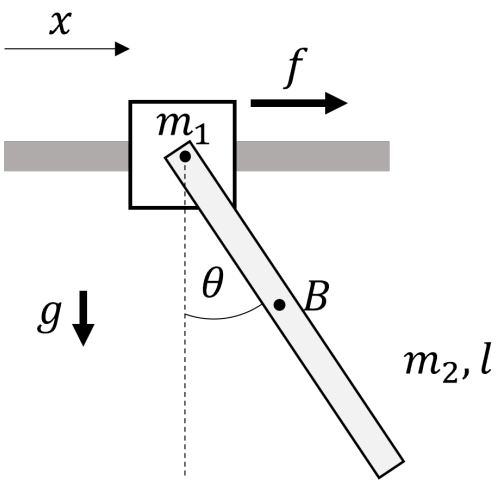



Fig. 3: Cart and pendulum

scalar to each vector.
Properties of a Norm:

1. Positive Definiteness: It should always be nonnegative. It is zero if and only if the vector is zero, i.e., zero vector. $\|v\| \geq 0$ and $\|v\| = 0 \Leftrightarrow v = 0$
2. Homogeneity: Multiplying a vector by a scalar multiplies the vector's norm by the scalar's absolute value. $\|\alpha v\| = |\alpha|\|v\|$
3. Triangle inequality: The norm of a sum of two vectors is no more than the sum of their norms. $\|v\| + \|u\| \geq \|v + u\|$

Two useful norms are

1. Norm-2 : $\|x\|_2 = \sqrt{\sum_{i=1}^{n} |x_i|^2}$
2. Norm-infinity: $\|x\|_\infty = \max |x_i|$

**Question 4.** Calculate $\|x\|_2$ and $\|x\|_\infty$ where

1. $x = [x_1, x_2, x_3]$ and $|x_2| \geq |x_1| \geq |x_3|$
2. $x = [1, 0, 0]$
3. $x = [1, 1, 1]$

**Question 5.** Find the sets of a 2D vector $x$ such that $\|x\|_2 = 1$ and $\|x\|_\infty = 1$