# Linear Systems (034032)
## lecture no. 4

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## Outline

Discrete-time frequency-domain analysis

A/D conversion in the frequency domain

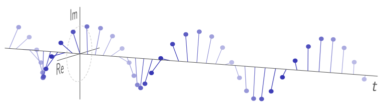
D/A conversion in the frequency domain

The Sampling Theorem



## Discrete harmonic signal

Signal $\alpha = a\exp_{e^{j\theta}} : \mathbb{Z} \to \mathbb{C}$,

$$\alpha[t] = a e^{j\theta t} = |a| e^{j(\theta t + \arg(a))} =$$ 


for $a \in \mathbb{C}$ and $\theta \in \mathbb{R}$ is called harmonic signal with frequency $\theta$, amplitude $|a|$, and initial phase $\arg(a)$. Given $T \in \mathbb{N}$,

$$\alpha[t+T] = a e^{j\theta(t+T)} = a e^{j\theta t + j\theta T} = a e^{j\theta t} e^{j\theta T} = \alpha[t]\, e^{j\theta T}$$

so that

$$\alpha[t+T] = \alpha[t] \iff e^{j\theta T} = 1 \iff |\theta| T = 2\pi k \quad \text{for } k \in \mathbb{N}$$

and this condition is independent of $t$. Therefore,

- $\alpha$ is $T$-periodic iff $\exists k \in \mathbb{N}$ such that $|\theta| = \dfrac{2\pi k}{T}$,

which happens iff $2\pi/|\theta| \in \mathbb{Q}$.



## Discrete harmonic signal (contd)

Because

$$e^{j(\theta+2\pi)t} = e^{j\theta t} e^{j2\pi t} = e^{j\theta t},$$

shifting $\theta$ by a multiple of $2\pi$ does *not* change the harmonic signal. Hence,

- discrete frequencies only make sense on intervals of the length $2\pi$.

Thereafter we use the convention that $\theta \in [-\pi, \pi]$, which is a choice.

The normalized change at each step of the harmonic signal,

$$\frac{|\alpha[t+1] - \alpha[t]|}{|\alpha[t]|} = \left| \frac{a e^{j\theta(t+1)}}{a e^{j\theta t}} - 1 \right| = |e^{j\theta} - 1| = 2\left|\sin\frac{\theta}{2}\right|,$$

is independent of $t$ and increases with $|\theta|$ (provided $\theta \in [-\pi, \pi]$, of course). It is thus still justifiable to say that

- $a_1\exp_{e^{j\theta_1}}$ is faster (slower) than $a_2\exp_{e^{j\theta_2}}$ if $|\theta_1| > |\theta_2|$ ($|\theta_1| < |\theta_2|$).

The fastest harmonic signal is that for $\theta = \pm\pi$, for which $\alpha[t] = a(-1)^t$.

## Discrete-time Fourier transform

The discrete-time Fourier transform (DTFT) of a discrete signal $x : \mathbb{Z} \to \mathbb{R}$ is the signal $X : [-\pi, \pi] \to \mathbb{C}$ such that

$$X(\mathrm{e}^{\mathrm{j}\theta}) = (\mathfrak{F}\{x\})(\mathrm{e}^{\mathrm{j}\theta}) := \sum_{t\in\mathbb{Z}} x[t]\mathrm{e}^{-\mathrm{j}\theta t}$$

where $\theta$ is called the angular frequency (in radians per step). The DTFT is well defined (uniform convergence, continuous $X$) if $x \in \ell_1$ and then

$$x[t] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\mathrm{e}^{\mathrm{j}\theta})\mathrm{e}^{\mathrm{j}\theta t}\mathrm{d}\theta =: (\mathfrak{F}^{-1}\{X\})(\mathrm{e}^{\mathrm{j}\theta})$$

If the domain of $X$ is extended to $\mathbb{R}$, then $X$ is a $2\pi$-periodic function of $\theta$.

If $x \in \ell_2$, then the DTFT sum might converge only in the (weaker) $\ell_2$-norm sense, so that $X$ is defined *almost everywhere* and might not be continuous or even bounded. The use of distributions facilitates extending the DTFT to yet wider classes of signals.



## Meaning

Similarly to the continuous-time case, the relation

$$x[t] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\mathrm{e}^{\mathrm{j}\theta})\mathrm{e}^{\mathrm{j}\theta t}\mathrm{d}\theta$$

means that

- $x$ is a superposition of elementary harmonics $\frac{1}{2\pi}X(\mathrm{e}^{\mathrm{j}\theta})\exp_{\mathrm{e}^{\mathrm{j}\theta}}$,

with a continuum of frequencies $\theta$, although now in the final range $[-\pi, \pi]$.

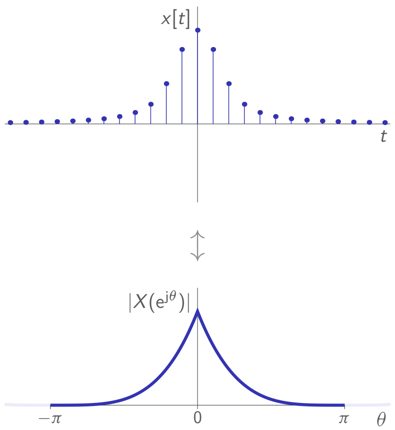
The signal $X$ is then called the spectrum or frequency-domain representation of $x$, with the amplitude spectrum $|X|$ and the phase spectrum $\arg(X)$.

The value $X(\mathrm{e}^{\mathrm{j}0}) = X(1)$ is the average of $x$ over all its domain, i.e. $\mathbb{Z}$.

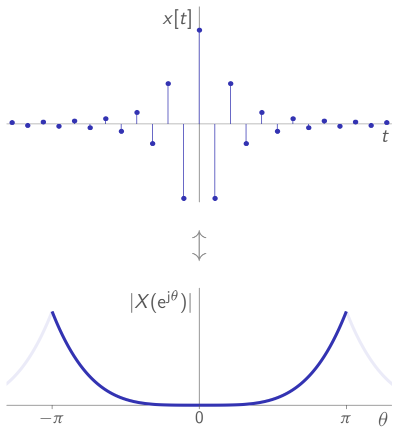


## Example

"slow" signal




"fast" signal






## Basic properties

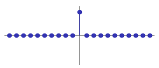
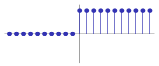
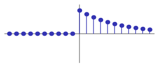
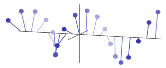
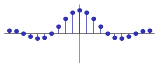
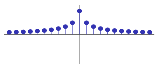
| property | time domain | frequency domain |
|---:|:---:|:---:|
| linearity | $x = a_1x_1 + a_2x_2$ | $X(\mathrm{e}^{\mathrm{j}\theta}) = a_1X_1(\mathrm{e}^{\mathrm{j}\theta}) + a_2X_2(\mathrm{e}^{\mathrm{j}\theta})$ |
| time shift | $y = \mathbb{S}_\tau x$ | $Y(\mathrm{e}^{\mathrm{j}\theta}) = \mathrm{e}^{\mathrm{j}\theta\tau}X(\mathrm{e}^{\mathrm{j}\theta})$ |
| time reversal | $y = \mathbb{P}_{-1}x$ | $Y(\mathrm{e}^{\mathrm{j}\theta}) = X(\mathrm{e}^{-\mathrm{j}\theta})$ |
| conjugation | $y = \overline{x}$ | $Y(\mathrm{e}^{\mathrm{j}\theta}) = \overline{X(\mathrm{e}^{-\mathrm{j}\theta})}$ |
| modulation | $y = x\exp_{\mathrm{e}^{\mathrm{j}\theta_0}}$ | $Y(\mathrm{e}^{\mathrm{j}\theta}) = X(\mathrm{e}^{\mathrm{j}(\theta-\theta_0)})$ |
| convolution | $z = x * y$ | $Z(\mathrm{e}^{\mathrm{j}\theta}) = X(\mathrm{e}^{\mathrm{j}\theta})Y(\mathrm{e}^{\mathrm{j}\theta})$ |

Parseval: if $x \in \ell_2$, then

$$\|x\|_2^2 = \sum_{t\in\mathbb{Z}}|x[t]|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi}|X(\mathrm{e}^{\mathrm{j}\theta})|^2\mathrm{d}\theta = \frac{1}{2\pi}\|X\|_2^2$$

Remember, $(\mathbb{S}_\tau x)[t] = x[t+\tau]$ and $(\mathbb{P}_{-1}x)[t] = x[-t]$.

## DTFTs of some discrete signals

| $x[t]$ | $X(e^{j\theta})$, $\theta \in [-\pi, \pi]$ | condition |
|---|---|---|
| $\delta[t] =$  | $1$ | |
| $\mathbb{1}[t] =$  | $\dfrac{1}{1 - e^{-j\theta}} + \pi\delta(\theta)$ | |
| $\lambda^t \mathbb{1}[t] =$  | $\dfrac{1}{1 - \lambda e^{-j\theta}}$ | $\lvert\lambda\rvert < 1$ |
| $e^{j\theta_0 t} =$  | $2\pi\, \mathrm{rect}_{2\pi}(\theta) \sum_{i \in \mathbb{Z}} \delta(\theta - \theta_0 - 2\pi i)$ | $\lvert\theta_0\rvert \le \pi$ |
| $\mathrm{sinc}[\pi\eta t] =$  | $\dfrac{1}{\eta} \mathrm{rect}_{2\pi\eta}[\theta]$ | $0 < \eta \le 1$ |
| $\dfrac{1}{1 + \lvert t\rvert} =$  | $-\ln(2 - 2\cos\theta)\cos\theta + (\pi - \lvert\theta\rvert)\sin\lvert\theta\rvert - 1$ | |



## Discrete Fourier transform

The discrete counterpart of the Fourier series is called the discrete Fourier transform (DFT). Namely, if $x : \mathbb{Z} \to \mathbb{R}$ is $T$-periodic, then at every $t$

$$x[t] = \frac{1}{T} \sum_{k=0}^{T-1} X[k] e^{j2\pi t k/T},$$

where

$$X[k] = \sum_{t=0}^{T-1} x[t] e^{-j2\pi k t/T}, \quad k \in \mathbb{Z}_{0..T-1}$$

Note that each harmonic in the DFT, $e^{j2\pi t k/T}$, is a $T$-periodic function of $t$. The DFT plays an important role in

- numerical algorithms (including numerical Fourier transforms, via FFT)
- image processing
- ...

but is less prominent in the context of linear systems studied in this course.



## Outline

Discrete-time frequency-domain analysis

A/D conversion in the frequency domain

D/A conversion in the frequency domain

The Sampling Theorem



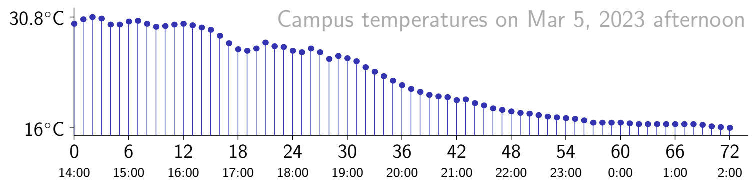
## The ideal sampler: what do we know

It maps continuous-time signals $x : \mathbb{R} \to \mathbb{R}$ to discrete signals $\bar{x} : \mathbb{Z} \to \mathbb{R}$ as

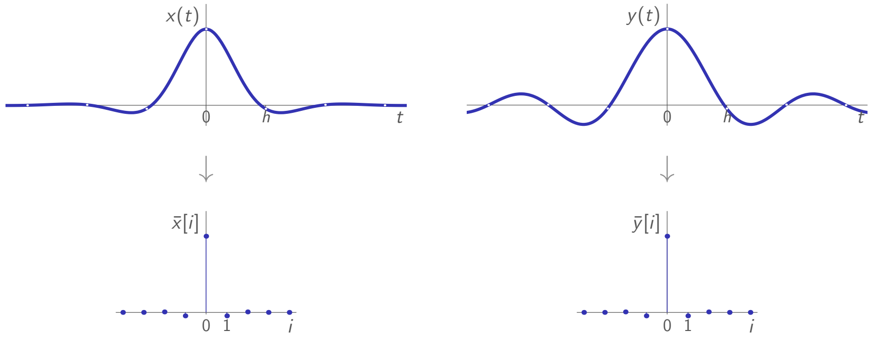
$$\bar{x}[i] = x(ih), \quad \forall i$$

for a given sampling period $h > 0$ (we assume periodic sampling hereafter). We may also think in terms of the sampling frequency $\omega_s := 2\pi/h$ (radians per time unit).

Sometimes it's convenient to think of $\bar{x}$ not over the abstract $\mathbb{Z}$, but rather over $\{\ldots, -2h, -h, 0, h, 2h, \ldots\}$, i.e. synchronized with the analog time $t$:



## Sampling: a key question



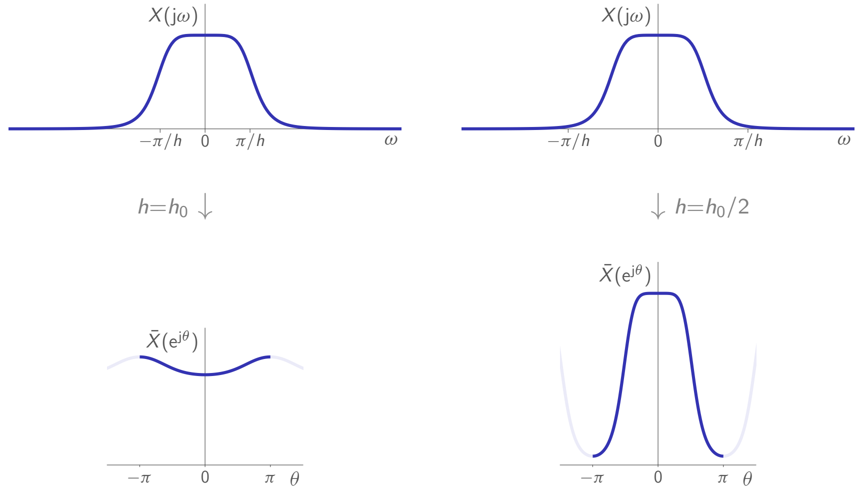

The question of

– what is lost in transforming the domain $\mathbb{R}$ to the domain $\mathbb{Z}$

might not be straightforward to address in the time domain. Is it apparent that sampling $x$ is problematic, whereas in sampling $y$ we loose nothing for the very same sampling period $h$?



## Sampling: a key question (contd)




Let's try to understand

– what is lost in transforming (squeezing) the domain $\mathbb{R}$ to $[-\pi, \pi]$.



## Periodic summation

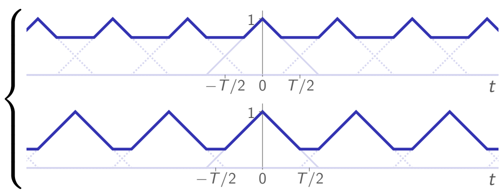
Consider a function $x : \mathbb{R} \to \mathbb{F}$, its periodic summation with period $T$ is

$$x_T := \sum_{i\in\mathbb{Z}} \mathbb{S}_{iT} x \quad\implies\quad x_T(t) = \sum_{i\in\mathbb{Z}} x(t + iT)$$

The function $x_T : \mathbb{R} \to \mathbb{F}$ is $T$-periodic, because

$$x_T(t+T) = \sum_{i\in\mathbb{Z}} x(t+iT+T)\Big|_{j=i+1} = \sum_{j\in\mathbb{Z}} x(t+jT) = x_T(t)$$

Example: if $x = \text{tent}$, then

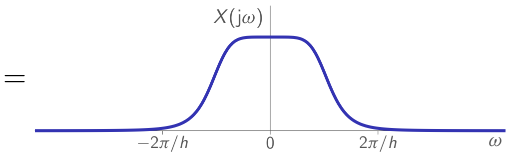
$x_T =$  if $T = \frac{4}{3}$; if $T = \frac{5}{3}$



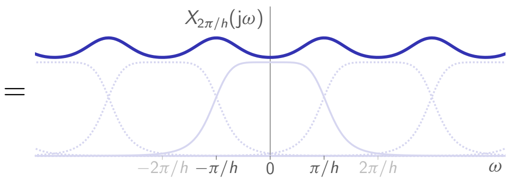


## Periodic summation of continuous-time spectra

Let $x$ be a continuous-time signal with the frequency response $X$, say

$X(j\omega) =$ 


and consider its periodic summation with the period $T = 2\pi/h$,

$X_{2\pi/h}(j\omega) = \sum_{i\in\mathbb{Z}} X(j(\omega + \frac{2\pi}{h} i)) =$ 


Because this function is periodic, it can be expanded into the Fourier series $X_{2\pi/h}(j\omega) = \sum_{k\in\mathbb{Z}} X_{2\pi/h}[k] e^{jhk\omega}$ (the fundamental frequency is $\omega_0 = h$).

## Fourier coefficients of $X_{2\pi/h}(\mathrm{j}\omega)$

By the definition of Fourier coefficient and taking into account that $\omega_0 = h$,

$$
\begin{aligned}
X_{2\pi/h}[k] &= \frac{1}{2\pi/h}\int_{-\pi/h}^{\pi/h} X_{2\pi/h}(\omega)\mathrm{e}^{-\mathrm{j}hk\omega}\,\mathrm{d}\omega \\
&= \frac{h}{2\pi}\int_{-\pi/h}^{\pi/h}\sum_{i\in\mathbb{Z}} X(\mathrm{j}(\omega+\tfrac{2\pi}{h}i))\mathrm{e}^{-\mathrm{j}kh\omega}\,\mathrm{d}\omega\,\underbrace{\mathrm{e}^{-\mathrm{j}2\pi ki}}_{=1} \\
&= \frac{h}{2\pi}\sum_{i\in\mathbb{Z}}\int_{-\pi/h}^{\pi/h} X(\mathrm{j}(\omega+\tfrac{2\pi}{h}i))\mathrm{e}^{-\mathrm{j}kh(\omega+2\pi i/h)}\,\mathrm{d}\omega\Big|_{\eta=\omega+2\pi i/h} \\
&= \frac{h}{2\pi}\sum_{i\in\mathbb{Z}}\int_{(2i-1)\pi/h}^{(2i+1)\pi/h} X(\mathrm{j}\eta)\mathrm{e}^{-\mathrm{j}\eta kh}\,\mathrm{d}\eta\Big|_{\omega=\eta} \\
&= \frac{h}{2\pi}\int_{\mathbb{R}} X(\mathrm{j}\omega)\mathrm{e}^{\mathrm{j}\omega(-kh)}\,\mathrm{d}\omega \qquad \text{cf. } x(t) = \frac{1}{2\pi}\int_{\mathbb{R}} X(\mathrm{j}\omega)\mathrm{e}^{\mathrm{j}\omega t}\,\mathrm{d}\omega \\
&= hx(-kh)
\end{aligned}
$$



## Fourier series of $X_{2\pi/h}(\mathrm{j}\omega)$

Thus

$$X_{2\pi/h}(\mathrm{j}\omega) = \sum_{k\in\mathbb{Z}} hx(-kh)\mathrm{e}^{\mathrm{j}hk\omega}\Big|_{i=-k} = h\sum_{i\in\mathbb{Z}} x(ih)\mathrm{e}^{-\mathrm{j}(h\omega)i}$$

At the same time, the DTFT of $\bar{x}$ such that if $\bar{x}[i] = x(ih)$ satisfies

$$\bar{X}(\mathrm{e}^{\mathrm{j}\theta}) = \sum_{i\in\mathbb{Z}}\bar{x}[i]\mathrm{e}^{-\mathrm{j}\theta i} = \sum_{i\in\mathbb{Z}} x(ih)\mathrm{e}^{-\mathrm{j}\theta i}$$

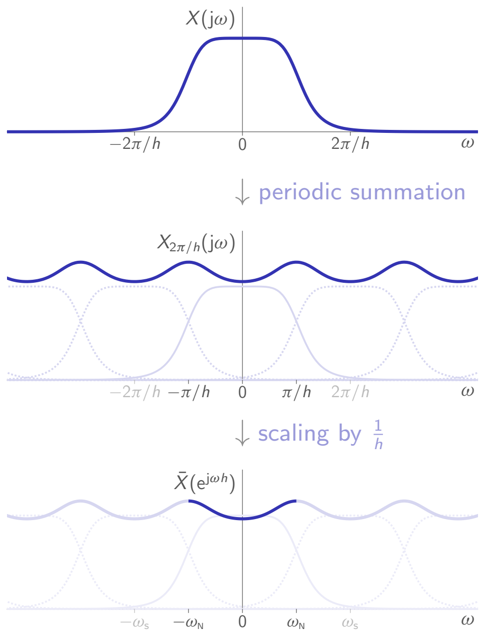
Hence, the DTFT of the sampled signal $\bar{x}$

$$\bar{X} = \frac{1}{h}\mathbb{P}_{1/h}X_{2\pi/h} \quad\Longrightarrow\quad \bar{X}(\mathrm{e}^{\mathrm{j}\theta}) = \frac{1}{h}X_{2\pi/h}(\mathrm{j}\tfrac{\theta}{h}) = \frac{1}{h}\sum_{i\in\mathbb{Z}} X(\mathrm{j}\tfrac{\theta+2\pi i}{h})$$

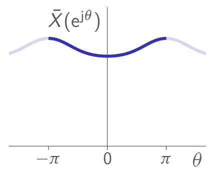
i.e. it's the periodic summation, whose period equals the sampling frequency $\omega_\mathrm{s}$, of the spectrum of the continuous-time $x$, scaled by the factor $1/h$ both in amplitude and in frequency.



## Spectrum of sampled signal




$\xrightarrow{\theta=\omega h}$




spectrum of $\bar{x}$:

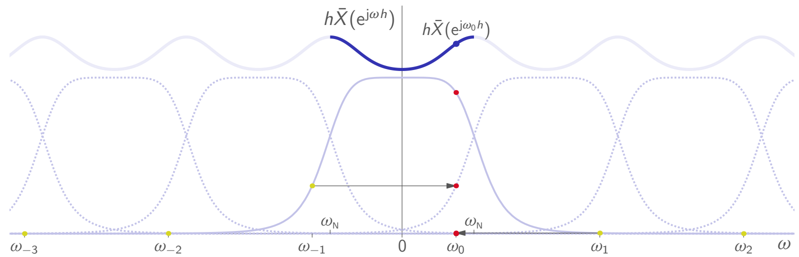
$$\bar{X}(\mathrm{e}^{\mathrm{j}\theta}) = \frac{1}{h}\sum_{i\in\mathbb{Z}} X(\mathrm{j}\tfrac{\theta+2\pi i}{h})$$

Nyquist frequency

$$\omega_\mathrm{N} := \frac{\pi}{h} = \frac{\omega_\mathrm{s}}{2}$$



## Spectrum of sampled signal: aliasing




The spectrum of $\bar{x}$ at each discrete frequency $\theta_0 = \omega_0 h$ is a

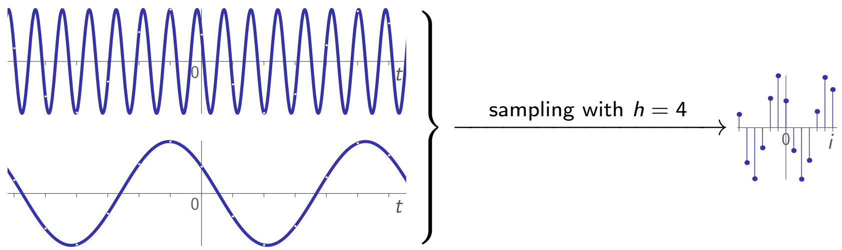
- blend of analog spectra at all analog frequencies $\omega_i := \omega_0 + \omega_\mathrm{s} i$, $i\in\mathbb{Z}$.

In this process every $\omega_i$ effectively tries to *alias* as $\theta_0$. This phenomenon is then dubbed aliasing, with respect to the base frequency $\omega_0$.

Aliasing means information loss, we can no longer tell $X(\mathrm{j}\omega_i)$ from $X(\mathrm{j}\omega_j)$ in their effect on $\bar{X}(\mathrm{e}^{\mathrm{j}\omega_0 h})$, unless we know how they depend on each other. For instance, in this particular case $\bar{X}(\mathrm{e}^{\mathrm{j}\omega_0 h})$ has significant contributions of both $X(\mathrm{j}\omega_0)$ and $X(\mathrm{j}\omega_{-1})$ for most $\omega_0 > 0$, which is problematic indeed.

## Aliasing: examples

Two sine waves: Remember (Lect. 2, Slide 35)



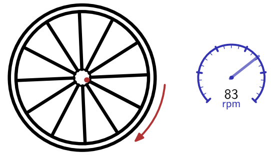

They are both of the form $\sin(\omega_i t + 2.6)$ for $\omega_1 = \frac{2\pi+1}{4}$ and $\omega_2 = \frac{1}{4}$. With $h = 4$ the dominant frequencies of their spectra alias as $\theta_0 = \omega_0 h = 1$:

$$\omega_1 = \omega_0 + 1 \cdot \frac{2\pi}{h} \quad \text{and} \quad \omega_2 = \omega_0 + 0 \cdot \frac{2\pi}{h}$$

That's why sampled versions of both these signals are the same, $\sin[t + 2.6]$.



## Aliasing: examples (contd)

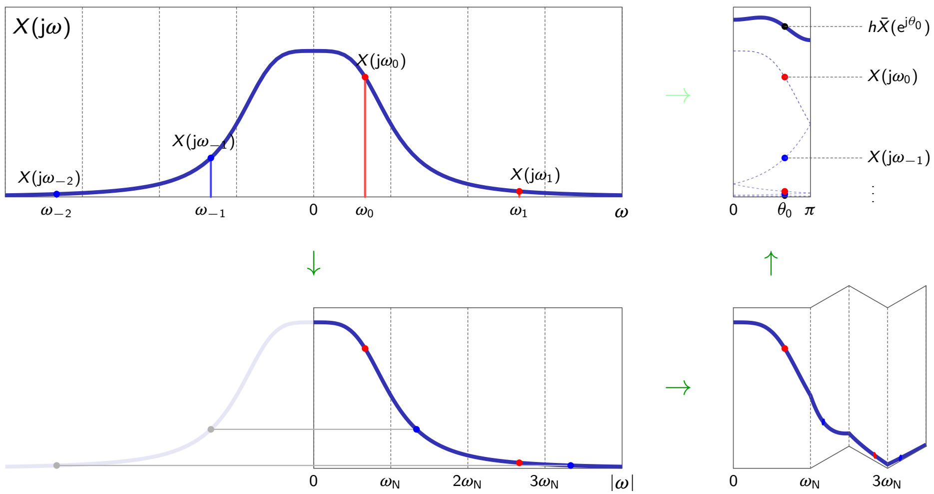
Wagon-wheel effect:  (shot with 12 FPS frame rate)


Moiré pattern: 




## Frequency folding

If $x$ is real-valued and even, then so is $X$ and $\bar{X}$ can be also constructed via the folding procedure:



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Discrete-time frequency-domain analysis

A/D conversion in the frequency domain

D/A conversion in the frequency domain

The Sampling Theorem

## The zero-order-hold: what do we know

It maps discrete signals $\bar{x} : \mathbb{Z} \to \mathbb{R}$ to continuous-time signals $x : \mathbb{R} \to \mathbb{R}$ as

$$x(t) = \bar{x}[i], \quad \forall t \in (ih, (i+1)h)$$

for a given sampling period $h > 0$.



## Spectrum of $x$

By the definition of the Fourier transform,

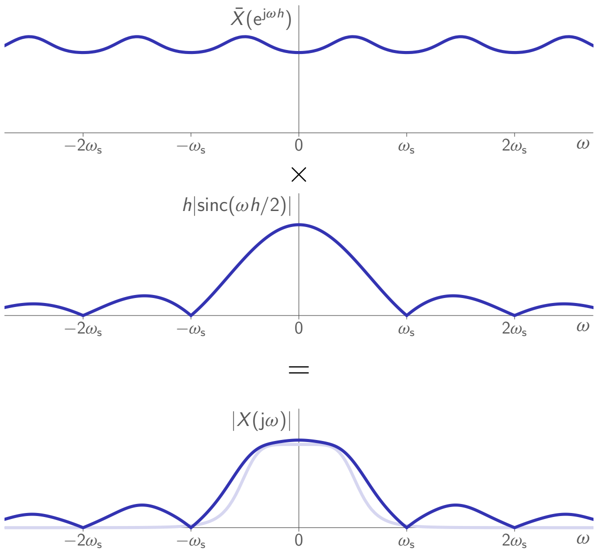
$$\begin{aligned}
X(\mathrm{j}\omega) &= \int_{\mathbb{R}} x(t)\mathrm{e}^{-\mathrm{j}\omega t}\mathrm{d}t = \sum_{i\in\mathbb{Z}} \int_{ih}^{(i+1)h} x(t)\mathrm{e}^{-\mathrm{j}\omega t}\mathrm{d}t \\
&= \sum_{i\in\mathbb{Z}} \int_{ih}^{(i+1)h} \bar{x}[i]\mathrm{e}^{-\mathrm{j}\omega t}\mathrm{d}t = \sum_{i\in\mathbb{Z}} \bar{x}[i] \int_{ih}^{(i+1)h} \mathrm{e}^{-\mathrm{j}\omega t}\mathrm{d}t \\
&= \sum_{i\in\mathbb{Z}} \bar{x}[i] \left(\frac{\mathrm{e}^{-\mathrm{j}\omega t}}{-\mathrm{j}\omega}\right)\Bigg|_{ih}^{(i+1)h} = \sum_{i\in\mathbb{Z}} \bar{x}[i] \frac{\mathrm{e}^{-\mathrm{j}\omega hi} - \mathrm{e}^{-\mathrm{j}\omega h(i+1)}}{\mathrm{j}\omega} \\
&= \sum_{i\in\mathbb{Z}} \bar{x}[i]\mathrm{e}^{-\mathrm{j}\omega hi} \frac{1-\mathrm{e}^{-\mathrm{j}\omega h}}{\mathrm{j}\omega} \qquad \text{cf. } \bar{X}(\mathrm{e}^{\mathrm{j}\theta}) = \sum_{i\in\mathbb{Z}} \bar{x}[i]\mathrm{e}^{-\mathrm{j}\theta i} \\
&= h\frac{1-\mathrm{e}^{-\mathrm{j}\omega h}}{\mathrm{j}\omega h}\bar{X}(\mathrm{e}^{\mathrm{j}\omega h}) = h\,\mathrm{sinc}\left(\frac{\omega h}{2}\right)\mathrm{e}^{-\mathrm{j}\omega h/2}\bar{X}(\mathrm{e}^{\mathrm{j}\omega h})
\end{aligned}$$

(note that $\bar{X}$ is $\omega_s$-periodic as a function of $\omega$).



## Example

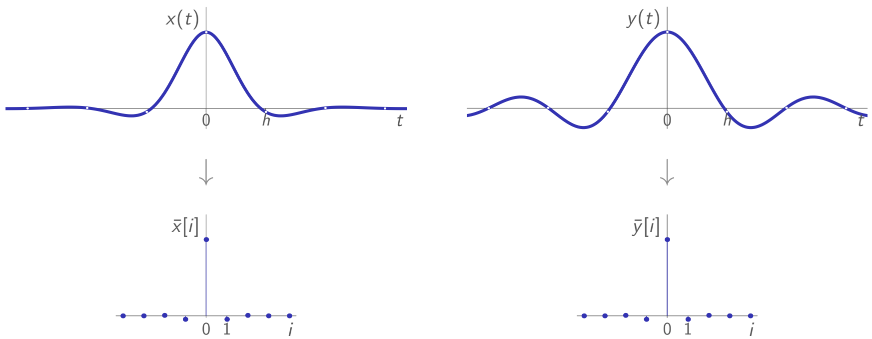
With $\bar{x}$ as above,






## Outline

## Sampling in the time domain (contd)



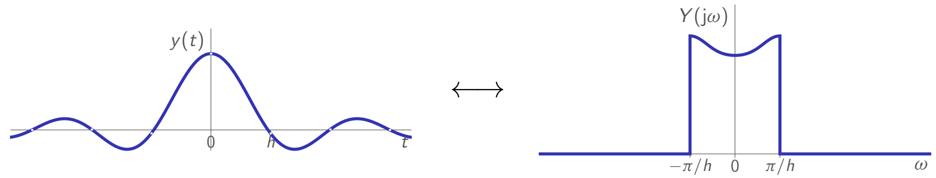

We saw why sampling of $x$ is problematic (strong aliasing, $\bar{X}$ is qualitatively different from $X$). But

- why do we loose nothing in sampling $y$?

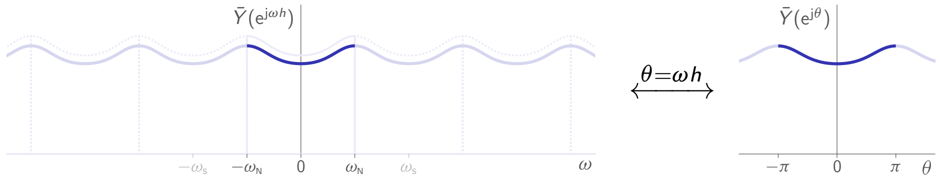


## Frequency-domain viewpoint

Spectrum of $y$ is




hence the spectrum of $\bar{y}$ is




has no contributions of $Y$ at aliased frequencies ($Y(\mathrm{j}\omega_i) = 0$ for all $i \neq 0$). As a result, $\bar{Y}$ is merely a scaled version of $Y$ and sampling is lossless here. This conclusion obviously applies to

- all signals whose spectrum has support within $[-\omega_N, \omega_N]$ (bandlimited).



## Reconstructing $x$ from its lossless sampling

If $\operatorname{supp}(X) \subset [-\omega_N, \omega_N]$, then $\bar{X}(\mathrm{e}^{\mathrm{j}\omega h}) = \frac{1}{h} X(\mathrm{j}\omega)$ for all $\omega \in [-\omega_N, \omega_N]$ and

$$\begin{aligned}
x(t) &= \frac{1}{2\pi} \int_{\mathbb{R}} X(\mathrm{j}\omega) \mathrm{e}^{\mathrm{j}\omega t} \mathrm{d}\omega = \frac{h}{2\pi} \int_{-\omega_N}^{\omega_N} \bar{X}(\mathrm{j}\omega) \mathrm{e}^{\mathrm{j}\omega t} \mathrm{d}\omega \\
&= \frac{1}{2\omega_N} \int_{-\omega_N}^{\omega_N} \sum_{i \in \mathbb{Z}} \bar{x}[i] \mathrm{e}^{-\mathrm{j}\omega h i} \mathrm{e}^{\mathrm{j}\omega t} \mathrm{d}\omega \\
&= \frac{1}{2\omega_N} \sum_{i \in \mathbb{Z}} \bar{x}[i] \int_{-\omega_N}^{\omega_N} \mathrm{e}^{-\mathrm{j}\omega h i} \mathrm{e}^{\mathrm{j}\omega t} \mathrm{d}\omega = \sum_{i \in \mathbb{Z}} \bar{x}[i] \frac{1}{2\omega_N} \left( \frac{\mathrm{e}^{\mathrm{j}(t-ih)\omega}}{\mathrm{j}(t-ih)} \right) \Bigg|_{-\omega_N}^{\omega_N} \\
&= \sum_{i \in \mathbb{Z}} \bar{x}[i] \frac{\mathrm{e}^{\mathrm{j}(t-ih)\omega_N} - \mathrm{e}^{-\mathrm{j}(t-ih)\omega_N}}{\mathrm{j}2(t-ih)\omega_N} = \sum_{i \in \mathbb{Z}} \bar{x}[i] \frac{\sin((t-ih)\omega_N)}{(t-ih)\omega_N} \\
&= \sum_{i \in \mathbb{Z}} \bar{x}[i] \operatorname{sinc}((t-ih)\omega_N)
\end{aligned}$$

In other words, signals that are bandlimited within $[-\omega_N, \omega_N]$ are uniquely described by their periodic samples.



## The Sampling Theorem

### Theorem (Whittaker-Kotel'nikov-Shannon)

*If* $\operatorname{supp}(X) \subset [-\omega_N, \omega_N]$, *then $x$ can be perfectly recovered from its sampled measurements as*

$$x = \sum_{i \in \mathbb{Z}} x(ih) \mathbb{S}_{-ih} \mathbb{P}_{\omega_N} \operatorname{sinc} \quad \implies \quad x(t) = \sum_{i \in \mathbb{Z}} x(ih) \operatorname{sinc}((t-ih)\omega_N)$$

*known as the sinc-interpolator* (sinc *hold).*

The sinc-interpolator acts as