Linear Systems (034032)
lecture no. 1

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Faculty of Mechanical Engineering
Technion—IIT

Outline

Outline

General info

Course web site: `http://leo.technion.ac.il/Courses/LS/`

Prerequisite (must): Algebra 1/Ext (104016) & ODE (104131)

- musts from Algebra 1:
  - complex numbers, vectors and matrices (including their basic operations), eigenvalues and eigenvectors, Jordan form, Cayley–Hamilton theorem
- musts from Calculus:
  - derivatives and integrals, power series
- musts from ODE:
  - solutions of first- and second-order differential equations

Grading policy:

- Quizzes during lectures (average of 5 best of 6, *magen*): 30%
  (provided the final exam grade is at least 55)
- Final exam: 70–100%

The use of books / lecture notes not permitted during exams.

Topics

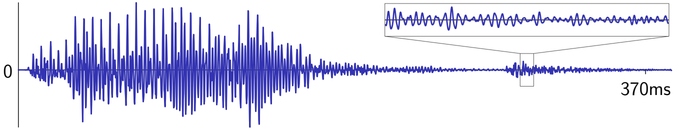
Main subjects (not necessarily in that order)

1. Intro: signals and systems
2. Signals and their properties
   - analog and digital signals in the time domain, A/D and D/A
   - signals in the frequency domain, Fourier analysis, A/D and D/A
   - signals in transformed domains (Laplace and z)
3. I/O systems
   - main properties (linearity, time-invariance, causality, stability)
   - LTI systems in the time domain (convolution representation)
   - LTI systems in the frequency domain
   - LTI systems in transformed domains (transfer functions)
4. State-space description of systems
   - state space and its properties
   - state equation solution, matrix exponential and other matrix functions
   - linearization

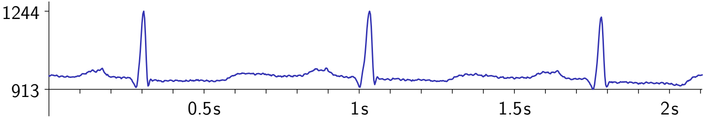
Outline

Signals: varying phenomena

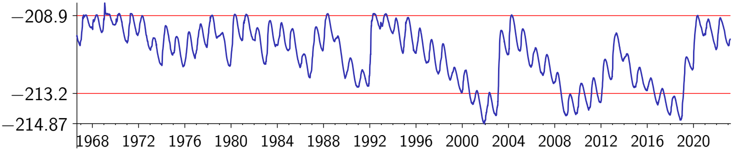
acoustic wave



ECG



Kinneret water level



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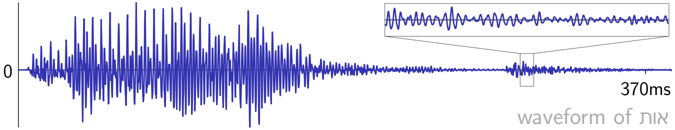

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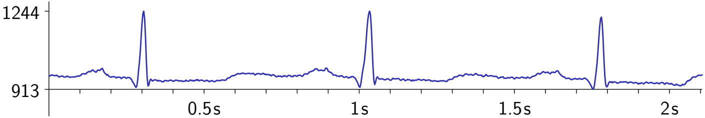

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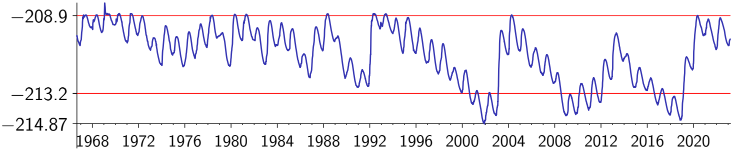

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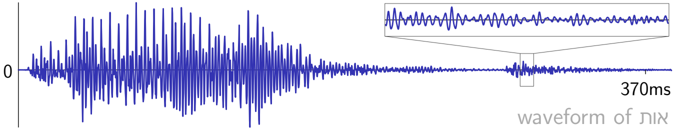

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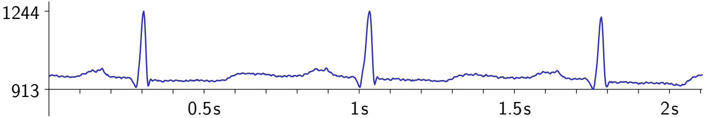

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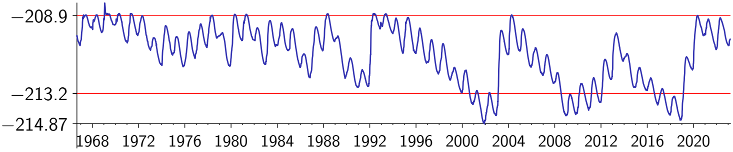

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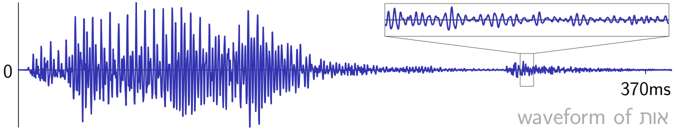

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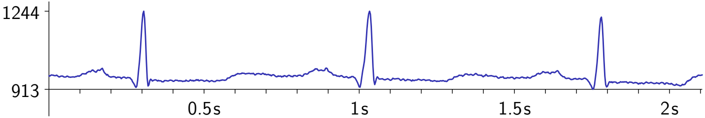

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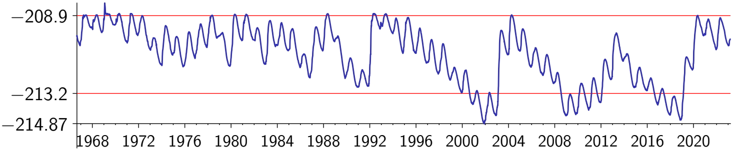
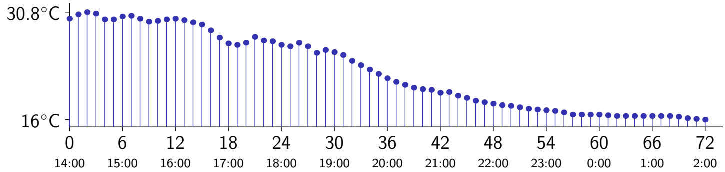
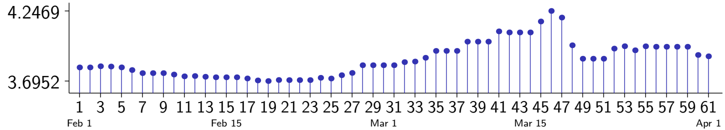


Signals: varying phenomena (contd)

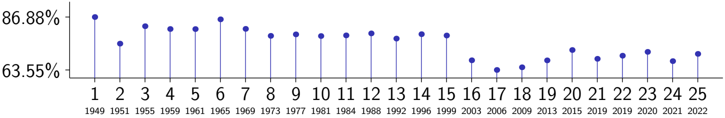
air temperature



currency exchange rate



elections turnout



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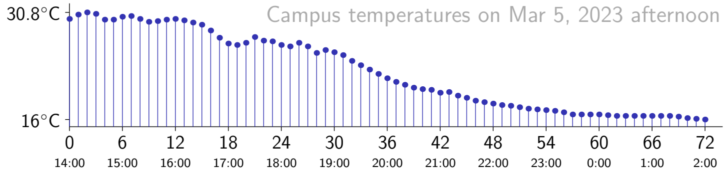

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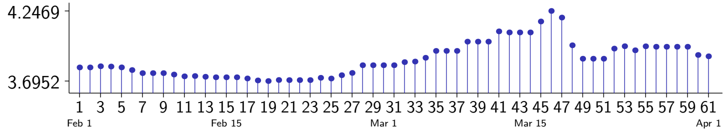

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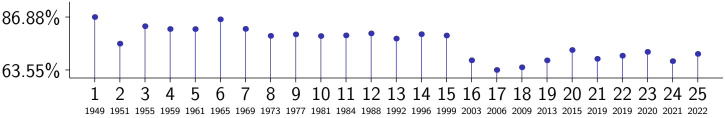

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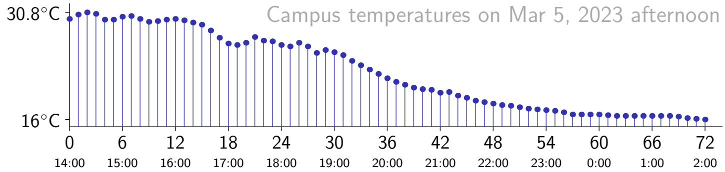

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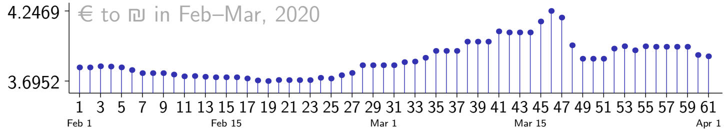

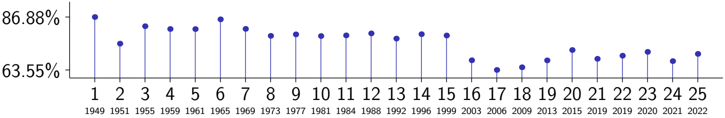

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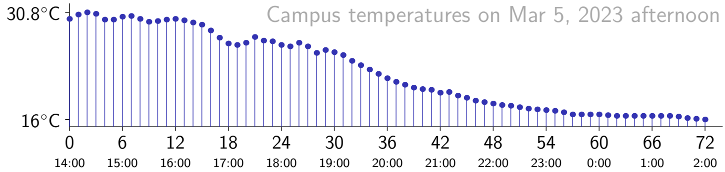

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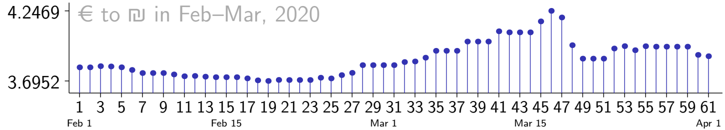

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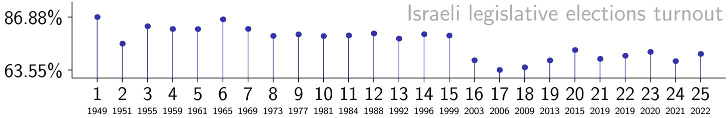
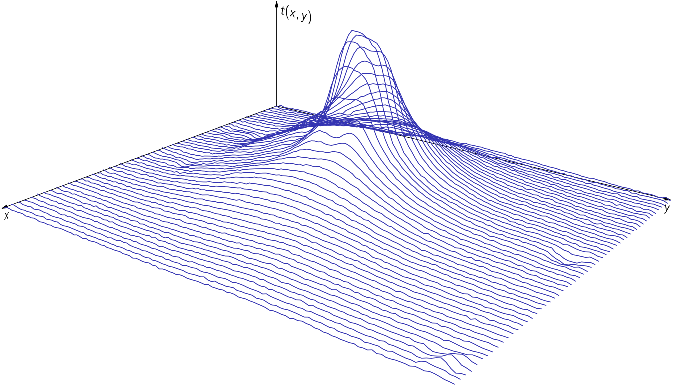


Signals: varying phenomena (contd)

steady-state temperature distribution



Systems

Signals are not isolated from each other, they are related via laws of nature, economics, society, and so forth. Sets of laws defining interactions between signals of interest are known as systems.

Systems may be viewed as

- constraints imposed on a *select set* of interacting signals of interest.

Our

- sensing
- decisions
- actions

are all determined by properties of corresponding systems. Comprehending these properties is thus of vital importance in all areas of engineering.

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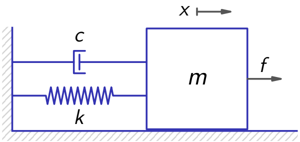
Systems: constraints on interdependent signals

mass-spring-damper system 1



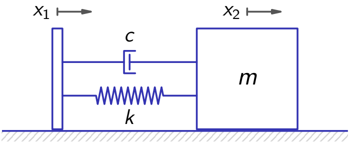
where

- mass position x [m]
- force f applied to the mass [N]

cannot change independently (think of spring scales, door dampers, etc).

Systems: constraints on interdependent signals (contd)

mass-spring-damper system 2



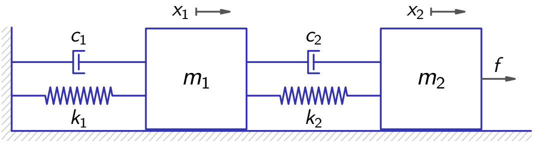
where

- plate position x_1
- mass position x_2

cannot change independently (think of car suspension, etc).

Systems: constraints on interdependent signals (contd)

mass-spring-damper system 3



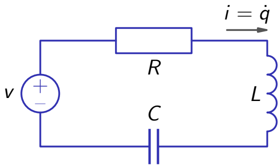
where

- positions x_1 of the first mass
- positions x_2 of the second mass
- force f applied to the second mass

cannot change independently.

Systems: constraints on interdependent signals (contd)

RLC circuit



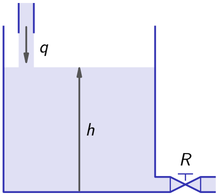
where

- current i (or charge q) [A] or [C]
- voltage v [V]

cannot change independently.

Systems: constraints on interdependent signals (contd)

liquid tank



where

- inflow rate q $[\mathrm{m}^3/\mathrm{s}]$
- liquid level h $[\mathrm{m}]$

cannot change independently.

Systems: constraints on interdependent signals (contd)

Other, more hand-waving, examples

- traffic: flow is linked with vehicle density
- financial: mortgage amortization payment is linked with the loan term
- economics: currency value is linked with the central bank interest rate
- networks: utility is linked with the size of large networks (Reed's law)
- biology: metabolic rate is related to animals' mass (Kleiber's law)
- ...

Course desired outcomes

Basic questions of the technological world are

1. how to understand existing systems?
2. how to affect properties of existing systems?
3. how to create new systems with desired properties?

This course is about the first question applied to a (relatively) simple class of systems, viz.

– linear (time-invariant) systems.

We shall learn how to describe and analyze them, connect and decompose, their main characteristics.

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Outline

All signals are equal, but some are more equal than others

In many situations

- some signals act $\leftarrow$ inputs
- other signals react $\leftarrow$ outputs

In this case it is convenient to view systems as

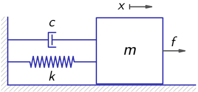
- mappings of input signals, say u, to output signals, say y, i.e. $u \mapsto y$

This viewpoint is known as input/output, or simply I/O, concept of systems (systems as operators or systems as signal processors) and assumes

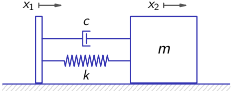
- directional information flow, from inputs to outputs, in systems.

I/O systems: examples

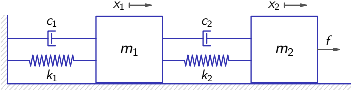
For previously studied systems,



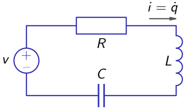
$$\implies \begin{cases} f \mapsto x & \text{if represents a spring scale} \\ x \mapsto f & \text{if represents a door damper} \end{cases}$$



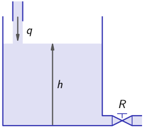
$$\implies x_1 \mapsto x_2$$



$$\implies f \mapsto \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$



$$\implies v \mapsto i \quad \text{or} \quad v \mapsto q$$



$$\implies \begin{cases} q \mapsto h & \text{if represents a pool} \\ h \mapsto q & \text{if represents a sprinkler} \end{cases}$$

Why I/O systems?

Sometimes, is the only way a system operates / supposed to operate

- radio receiver: the sound is caused by the broadcast signal, but cannot affect it back
- saving program: the income is caused by the deposit amount, interest rate, taxation, etc, but income affects none of those
- sensors: expected to measure w/o affecting measured phenomena

Sometimes, is an outcome of functionality

- electromechanical converter
  - motor, if "electrical" $\mapsto$ "mechanical"
  - generator, if "mechanical" $\mapsto$ "electrical"

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I/O systems: notation

A system $G : u \mapsto y$ will be written as

$$y = Gu$$

(**not** to be understood as a **product**).

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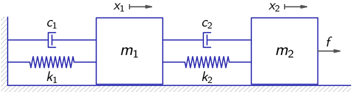
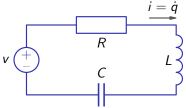
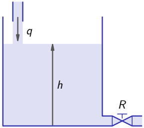


$$\Longrightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = G_2 f$$



$$\Longrightarrow \quad i = G_i v \quad \text{or} \quad q = G_q v$$

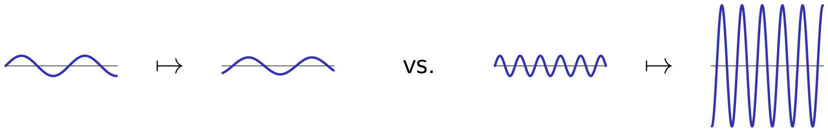


$$\Longrightarrow \begin{cases} h = G_{\text{pool}}\, q & \text{if a pool} \\ q = G_{\text{sprinkle}}\, h & \text{if a sprinkler} \end{cases}$$

Outline

Different nature, similar properties: resonance

An excitation at certain frequency produces disproportionally large response:



- Broughton Suspension Bridge & Pont de la Basse-Chaîne
  collapsed, on Apr 12, 1831 and Apr 16, 1850, when solders were marching across
- Tacoma Narrows Bridge
  collapsed on Nov 7, 1940 when twisting mode occurred from winds at 64 km/h
- acoustic wave breaks a glass
  a wine glass shatters if a person sings loudly and at the right tone near it
- radio / TV receiver
  extracts required channel literally "out of thin air"
- microwave oven
  food molecules vibrate when excited by emitted radiation at certain frequencies
- ...

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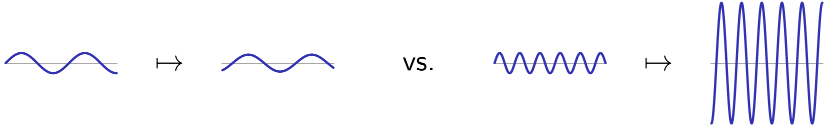

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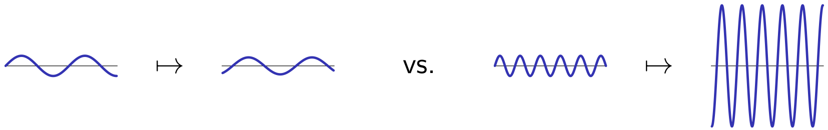

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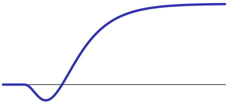
- . . .

Different nature, similar properties: inverse response

The initial response is opposite to the long-term response:



- gaining attitude by an aircraft
  tilting an elevator upward reduces the net lift
- gaining body energy via meals
  chewing and digestion (not to mention hunting / shopping) consume energy
- productivity via hiring new employees
  productivity initially decreases due to the training required
- balancing a pole on a hand
  to move the pole left, start from moving it right for a while
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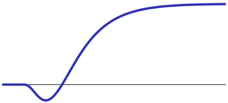

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Signals as functions of independent variable(s)

As usual, abstract mathematical thinking helps to find common ground . . .

Mathematically, signals are functions assigning to each element from their domains one element from their codomains.

Some domains:

- continuous time ($\mathbb{R}$, $\mathbb{R}_+$, $[t_1, t_2]$, . . .)
- discrete time ($\mathbb{Z}$, $\mathbb{Z}_+$, $\mathbb{N}$, $\mathbb{Z}_{t_1..t_2} := \{t \in \mathbb{Z} \mid t_1 \leq t \leq t_2\}$, . . .)
- space ($\mathbb{R}$, $\mathbb{R}^n$, $[x_1, x_2] \times [y_1, y_2]$, $\mathbb{Z}_{i_1..i_2} \times \mathbb{Z}_{j_1..j_2}$, even $\mathbb{R}_+ \times [x_1, x_2]$, . . .)
- transformed variables (j$\mathbb{R}$, $\mathbb{T} := \{z \in \mathbb{C} \mid |z| = 1\}$, $\mathbb{C}$, $\mathbb{C}^n$, . . .)

Some codomains:

- real or complex scalars ($\mathbb{R}$ or $\mathbb{C}$)
- real or complex vectors ($\mathbb{R}^n$, $\mathbb{C}^n$, or $\mathbb{R}^{n_1} \times \mathbb{C}^{n_2}$)
- binary scalars or vectors ($\{0, 1\}$ or $\{0, 1\}^n$)
- qbits ($\{|0\rangle, |1\rangle\}$)

Signals as functions of independent variable(s): notation

Signals are normally denoted by lowercase letters, u, v, w, ... If we write

$$u : \mathbb{R}_+ \to \mathbb{R}, \quad v : \mathbb{Z} \to \mathbb{C}^2, \quad w : \mathbb{Z}_{0..2047} \times \mathbb{Z}_{0..1279} \to [0,1]^3$$

we mean that

- u assigns to every element of $\mathbb{R}_+$ (domain) an element of $\mathbb{R}$ (codomain)
- v assigns to every element of $\mathbb{Z}$ an element of $\mathbb{C}^2$
- w assigns to every element of the 2048×1280 plane an RGB pixel

Continuous-time signals — domain is a subset of $\mathbb{R}$

- $u(t)$ means the value of u at a specific $t \in \mathbb{R}_+$, i.e. $u(t) \in \mathbb{R}$

Discrete-time signals — domain is a subset of $\mathbb{Z}$

- $v[t]$ means the value of v at a specific $t \in \mathbb{Z}$, i.e. $v[t] \in \mathbb{C}^2$
- $w[x, y]$ means the value of w at specific $x \in \mathbb{Z}_{0..2047}$ and $y \in \mathbb{Z}_{0..1279}$

Signals as functions of independent variable(s): notation

Signals are normally denoted by lowercase letters, u, v, w, ... If we write

$$u : \mathbb{R}_+ \to \mathbb{R}, \quad v : \mathbb{Z} \to \mathbb{C}^2, \quad w : \mathbb{Z}_{0..2047} \times \mathbb{Z}_{0..1279} \to [0,1]^3$$

we mean that

- u assigns to every element of $\mathbb{R}_+$ (domain) an element of $\mathbb{R}$ (codomain)
- v assigns to every element of $\mathbb{Z}$ an element of $\mathbb{C}^2$
- w assigns to every element of the 2048×1280 plane an RGB pixel

Continuous-time signals — domain is a subset of $\mathbb{R}$

- $u(t)$ means the value of u at a specific $t \in \mathbb{R}_+$, i.e. $u(t) \in \mathbb{R}$

Discrete-time signals — domain is a subset of $\mathbb{Z}$

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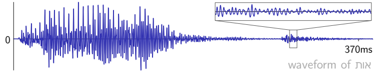
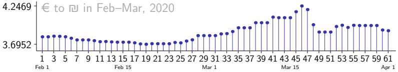
Signals as functions of independent variable(s): examples



$: \mathbb{R} \to \mathbb{R}$



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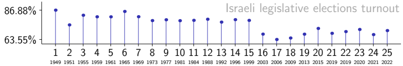

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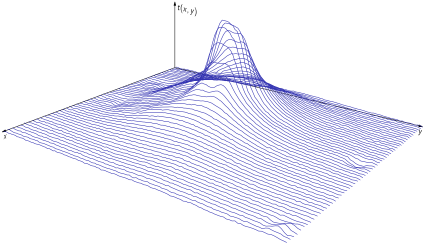

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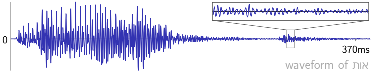

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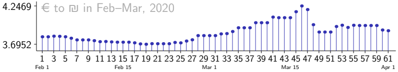

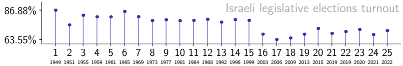

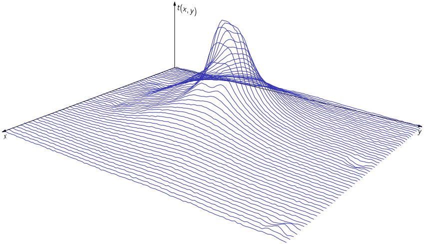

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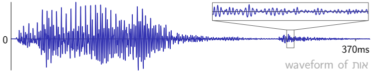

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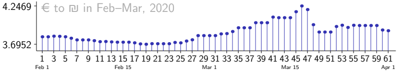

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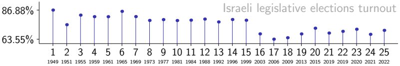

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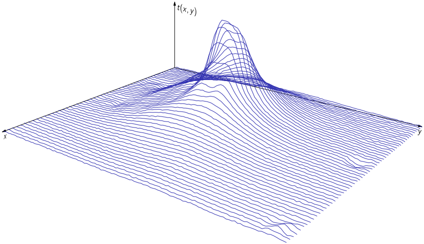

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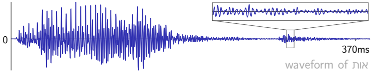

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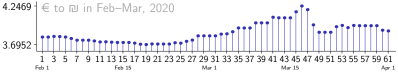

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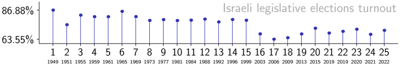

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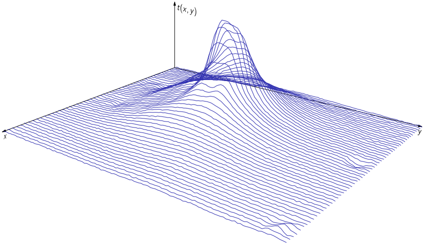

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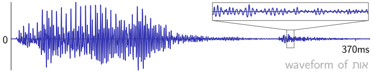

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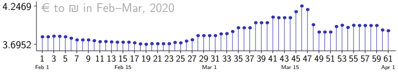

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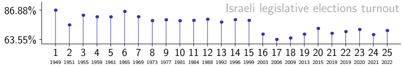

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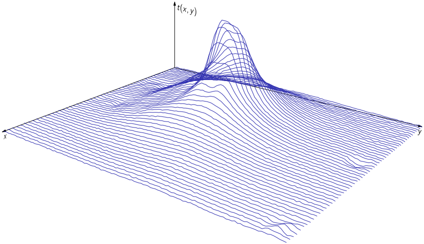

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Systems as (mathematical) models

Systems are normally represented by their models, which express

- relations between involved signals in an abstract (math) language.

Such relations are derived under simplifying assumptions about systems and their operation conditions and are thus truthful only to a certain extent and under certain conditions. Models are derived

- from first principles *ab initio*
- phenomenologically
- from observing experimental I/O relations *identification*

or combinations of those.

We often say "system" meaning its model. This is a common practice, but it's important to remember that

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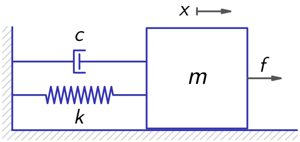
First principles modeling: mass-spring-damper 1



Assumptions:

- spring force is proportional to position differences (Hooke's law)
- dashpot force is proportional to velocity differences (viscous damping)
- we may *neglect* friction, force misalignment, etc

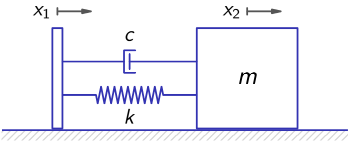
Supposing zero spring and dashpot forces at $x = 0$, by Newton's second law

$$m\ddot{x}(t) = f(t) + f_{\text{spring}}(t) + f_{\text{damp}}(t) = f(t) - kx(t) - c\dot{x}(t)$$

or, equivalently,

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = f(t).$$

First principles modeling: mass-spring-damper 2



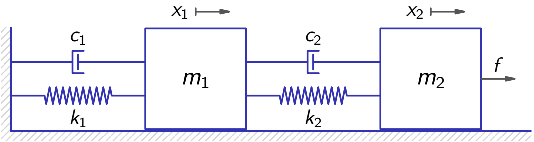
Under the same assumptions and supposing zero spring and dashpot forces at $x_1 = 0$ and $x_2 = x_{20} > 0$, by Newton's second law

$$m\ddot{x}_2(t) = f_{\text{spring}}(t) + f_{\text{damp}}(t) = -k\big(x_2(t) - x_{20} - x_1(t)\big) - c\big(\dot{x}_2(t) - \dot{x}_1(t)\big)$$

or, equivalently,

$$m\ddot{x}_2(t) + c\dot{x}_2(t) + kx_2(t) = c\dot{x}_1(t) + kx_1(t) + kx_{20}.$$

First principles modeling: mass-spring-damper 3



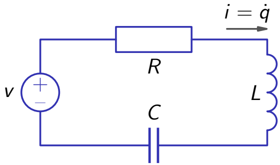
Under the same assumptions, by Newton's second law

$$m_1\ddot{x}_1(t) = k_2\big(x_2(t) - x_{20} - x_1(t)\big) + c_2\big(\dot{x}_2(t) - \dot{x}_1(t)\big) - k_1 x_1(t) - c_1\dot{x}_1(t)$$
$$m_2\ddot{x}_2(t) = f(t) - k_2\big(x_2(t) - x_{20} - x_1(t)\big) - c_2\big(\dot{x}_2(t) - \dot{x}_1(t)\big)$$

or, equivalently,

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{x}_1(t) \\ \ddot{x}_2(t) \end{bmatrix} + \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix}\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} -k_2 x_{20} \\ f(t) + k_2 x_{20} \end{bmatrix}.$$

First principles modeling: RLC circuits



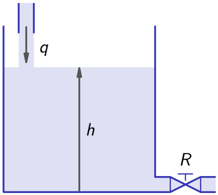
Assumptions:

- the resistor is ohmic (i.e. obeys Ohm's law) and linear
- the inductor obeys Faraday's law of induction
- capacitor charge q is proportional to the voltage across it
- we may *neglect* heat effects, etc

Kirchhoff's voltage law states that $v = v_R + v_L + v_C$, i.e.

$$L\ddot{q}(t) + R\dot{q}(t) + \frac{1}{C}q(t) = v(t).$$

First principles modeling: liquid tank



Assumptions:

- liquid is incompressible
- we may *neglect* the liquid viscosity

Mass balance and Torricelli's law:

$$\frac{\mathrm{d}}{\mathrm{d}t}(\rho A h(t)) = \rho q(t) - \rho q_{\mathrm{out}}(t) = \rho q(t) - \rho R\sqrt{h(t)}$$

with liquid density ρ, tank cross-sectional area A, valve resistance R. Thus,

$$A\dot{h}(t) + R\sqrt{h(t)} = q(t).$$

Phenomenological modeling: SIR epidemic spread model

Let

- s be the number of susceptibles in the population
- i be the number of infectives in the population
- r be the number of removed in the population

The SIR model:

$$\begin{cases} \frac{\mathrm{d}}{\mathrm{d}t}s(t) = -bs(t)i(t) & s(0) = s_0 \quad \text{(total population)} \\ \frac{\mathrm{d}}{\mathrm{d}t}i(t) = bs(t)i(t) - ai(t) & i(0) = 1 \quad \text{(patient zero, input)} \\ \frac{\mathrm{d}}{\mathrm{d}t}r(t) = ai(t) & r(0) = 0 \end{cases}$$

for some infection rate $b > 0$ and recovery rate $a > 0$.

Outline

Course info

Intro: signals & systems

I/O systems

Mathematical language

System interconnections

Open systems

The universe is the ultimate system, where all existing signals interact. But its properties would be

- hard (perhaps, impossible) to comprehend.

Understanding small processes, with a limited number of interacting signals, may be feasible. But

- real-world systems are complex.

A solution is to

- let systems interact with other systems via common signals.

Providing for such interaction interfaces makes systems open to the outside world. Hence, the term open systems.

Although this course doesn't get deep into interaction issues, we do

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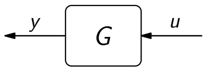
Block-diagrams

Information flow in I/O systems can be represented via block-diagrams, like



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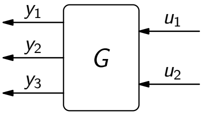

or

where

- lines represent signals — continuous-time $\xleftarrow{v}$ or discrete-time $\xleftarrow{v}$
- blocks represent systems — arrows show inputs and outputs

One system has special notation, viz.

- summation element — could have more than 2 inputs

: $y = u_1 + u_2$ or : $y = u_1 - u_2$

Block-diagrams are convenient for

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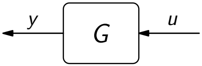

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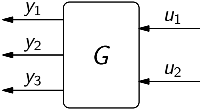

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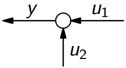

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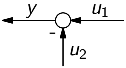

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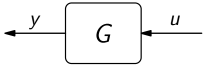

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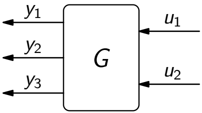

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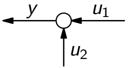

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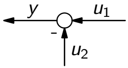

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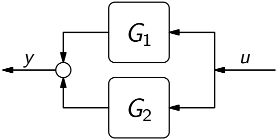

G_1 and G_2 have a common input and their outputs sum up. Notation:

$$y = G_1 u + G_2 u = (G_1 + G_2)u \quad \Longrightarrow \quad \underbrace{G = G_1 + G_2}_{\text{not a sum}}$$

spring-damper ($G : x \mapsto f_{\text{s-d}}$)

$$\underbrace{G_1 : x \mapsto f_{\text{spring}}, \quad G_2 : x \mapsto f_{\text{damp}}}$$

$$\Downarrow$$

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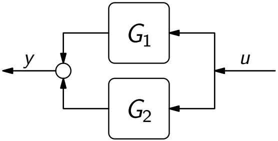

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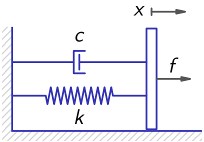

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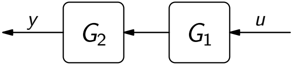
Series (cascade)



The output of G_1 is the input of G_2. Notation:

$$y = G_2(G_1 u) = G_2 G_1 u \quad \Longrightarrow \quad \underbrace{G = G_2 G_1}_{\text{not a product}}$$

2-tank system ($G : q_{\text{in}} \mapsto q_{\text{out}}$)

$$\underbrace{G_1 : q_{\text{in}} \mapsto q_{12}, \quad G_2 : q_{12} \mapsto q_{\text{out}}}$$

$$\Downarrow$$

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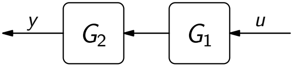
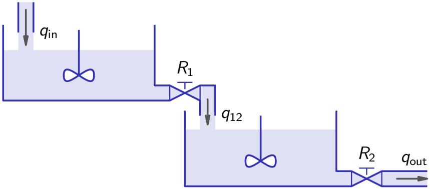


The output of G_1 is the input of G_2. Notation:

$$y = G_2(G_1 u) = G_2 G_1 u \quad \Longrightarrow \quad \underbrace{G = G_2 G_1}_{\text{not a product}}$$

2-tank system ($G : q_{\text{in}} \mapsto q_{\text{out}}$)

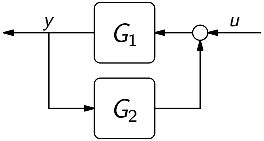


$$\underbrace{G_1 : q_{\text{in}} \mapsto q_{12}, \quad G_2 : q_{12} \mapsto q_{\text{out}}}$$

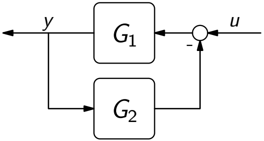
$$\Downarrow$$

$$\overbrace{G : q_{\text{in}} \mapsto q_{\text{out}}}$$

Feedback



or



The output of G_1 is the input of G_2 and the output of G_2 sums up with the exogenous input to produce the input to G_1.

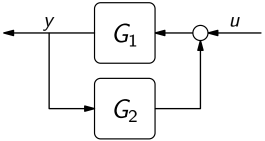
mass-spring-damper ($G : f \mapsto x$) may be seen as an interconnection of the Newtonian G_1 (mass × acceleration = net force) and the spring-damper G_2, on which the net force f_{net} depends and which itself depends on the output of G_1

$$G_1 : f_{\text{net}} \mapsto x, \quad G_2 : x \mapsto f_{\text{s-d}}, \quad f_{\text{net}} = f + f_{\text{s-d}}$$

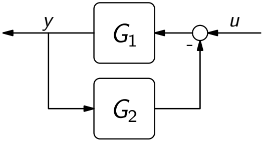
$$\Downarrow$$

$$G : f \mapsto x$$

Feedback



or



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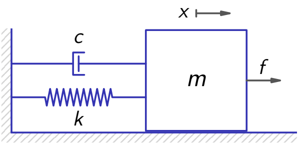

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$$\Downarrow$$

$$\overbrace{G : f \mapsto x}$$