

Linear Control Systems (036012)

lecture no. 1

Leonid Mirkin

Faculty of Mechanical Engineering
Technion — IIT



1/30

Outline

Course info

Introduction

Review of signals and systems

Review of control principles

Naïve MIMO

2/30

Course info

- Credit points: **3**
- Prerequisite: **Control Theory** (035188)
- Grading policy: homework **100%** (4 best out of 5 assignments)
Homework solutions must be submitted **electronically** to c036012@technion.ac.il
- Course site: <http://leo.technion.ac.il/Courses/LCS/>
- Literature:
 1. My **lecture notes** (available at the course site)
 2. Skogestad, S. & I. Postlethwaite. *Multivariable Feedback Control: Analysis and Design*, John Wiley & Sons, 1996.
 3. Doyle, J. C., B. A. Francis, & A. Tannenbaum. *Feedback Control Theory*, MacMillan, 1992 (available online).
 4. Zhou, K., J. C. Doyle, & K. Glover. *Robust and Optimal Control*, Prentice Hall, 1995.

3/30

Syllabus

1. Stand-alone systems

- static MIMO systems
- dynamic MIMO systems
 - basic notions (stability, causality, domain) in time and transformed domains
 - coprime factorization in H_∞
 - poles and zeros of rational transfer functions
 - state-space realizations and their structural and computational properties
 - model order reduction via balanced truncation

2. Interconnected systems

- basic interconnections and their effects on dynamics, LFTs
- stability and stabilization
 - internal stability
 - general stability results (Small Gain and Passivity)
 - all stabilizing controllers (Youla–Kučera)
- optimization-based performance
 - weighted / mixed sensitivity problems, the standard problem
 - balanced sensitivity (H_∞ loop shaping)

4/30

Outline

Course info

Introduction

Review of signals and systems

Review of control principles

Naïve MIMO

5/30

Why and for whom

Course goals

1. MIMO literacy (badly lacking, especially in industry)
2. Power and limitations of optimization-based design methods

Background needed

- linear algebra (see Appendix A of Lecture Notes)
- SISO systems
- classical SISO control methods

Hazard (or opportunity, depends on preferences)

- analytic material (no choice, hand-waving hurts in MIMO more)

6/30

A quiz

What in the order and what are poles and zeros of the transfer matrices:

1. $G(s) = \begin{bmatrix} 1/s & 0 \\ 0 & 1/s \end{bmatrix}$

2. $G(s) = \begin{bmatrix} 1/s & 1/s \\ 1/s & 1/s \end{bmatrix}$

3. $G(s) = \begin{bmatrix} 1 & 1/s \\ 0 & 1 \end{bmatrix}$

7/30

Nomenclature

\mathbb{R}	the set of real numbers, $\mathbb{R} = (-\infty, \infty)$
\mathbb{R}_+	the set of nonnegative real numbers, $\mathbb{R}_+ = [0, \infty)$
\mathbb{R}_-	the set of nonpositive real numbers, $\mathbb{R}_- = (-\infty, 0]$
$j\mathbb{R}$	the set of pure imaginary numbers
\mathbb{C}	the set of complex numbers
\mathbb{C}_α	the half plane to the right of $\alpha \in \mathbb{R}$, i.e. $\mathbb{C}_\alpha: \{z \in \mathbb{C} \mid \operatorname{Re} z > \alpha\}$
$\bar{\mathbb{C}}_\alpha$	the closure of \mathbb{C}_α , i.e. $\bar{\mathbb{C}}_\alpha: \{z \in \mathbb{C} \mid \operatorname{Re} z \geq \alpha\}$
\mathbb{T}	the unit circle, $\mathbb{T} := \{z \in \mathbb{C} \mid z = 1\}$
\mathbb{D}_α	the open α -disk, $\mathbb{D} := \{z \in \mathbb{C} \mid z < \alpha\}$
$\bar{\mathbb{D}}_\alpha$	the closed α -disk, $\bar{\mathbb{D}} := \{z \in \mathbb{C} \mid z \leq \alpha\} = \mathbb{D}_\alpha \cup (\alpha \mathbb{T})$
\mathbb{F}	alias of either \mathbb{R} or \mathbb{C}
\mathbb{Z}	the set of integers
\mathbb{N}	the set of positive integers (natural numbers)
\mathbb{Z}_+	the set of nonnegative integers
\mathbb{Z}_-	the set of nonpositive integers, $\mathbb{Z}_- = \mathbb{Z} \setminus \mathbb{N}$
$\mathbb{Z}_{i_1..i_2}$	the interval $\{i_1, i_1 + 1, \dots, i_2\}$

8/30

Outline

Course info

Introduction

Review of signals and systems

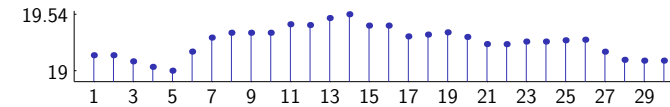
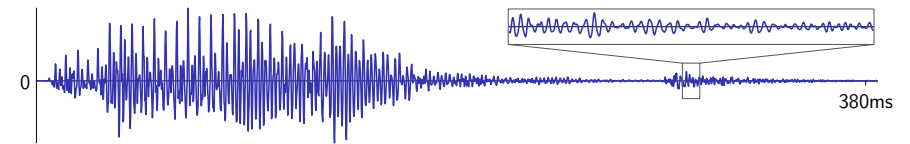
Review of control principles

Naïve MIMO

9/30

Signals

Represent **evolving information**:



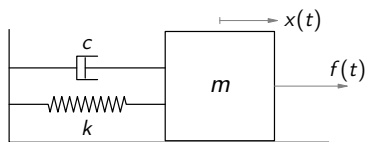
Mathematically,

- functions of independent variables, $f(t)$ or $f[t]$

10/30

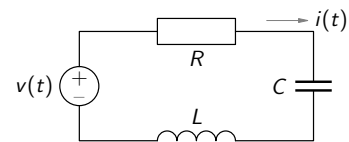
Systems

Constraints imposed on signals:



position $x(t)$ and force $f(t)$

$$x \mapsto f \text{ or } f \mapsto x$$



current $i(t)$ and voltage $v(t)$

$$v \mapsto i \text{ or } i \mapsto v$$

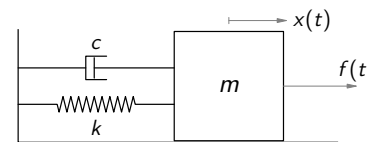
I/O view on systems:

- some signals act (**inputs**)
- some signals react (**outputs**)

11/30

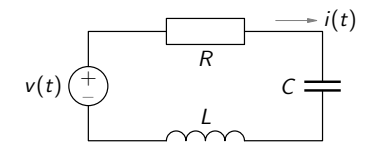
Mathematical models

(Approximate) description in a **mathematical language**:



position and force linked as

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = f(t)$$



charge ($\dot{q} = i$) and voltage linked as

$$L\ddot{q}(t) + R\dot{q}(t) + \frac{1}{C}q(t) = v(t)$$

Abstract form:

$$\ddot{y}(t) + 2\zeta\omega_n\dot{y}(t) + \omega_n^2y(t) = k_{st}\omega_n^2u(t)$$

12/30

Outline

Course info

Introduction

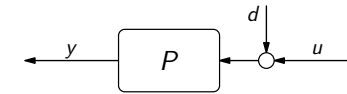
Review of signals and systems

Review of control principles

Naïve MIMO

13/30

Prototype control problem



y : regulated signal

u : control signal (means)

d : load disturbance

P : plant

Goal:

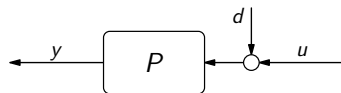
$$u \longrightarrow y = r$$

where

r : reference signal (goal)

14/30

Ultimate methodology: plant inversion



$$y = P(d + u) \wedge y = r$$

\Downarrow

$$r = P(d + u)$$

\Downarrow

$$u = P^{-1}r - d$$

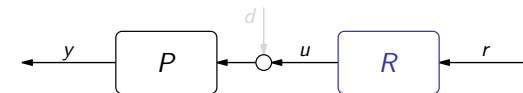
where

— P^{-1} is the inverse system

defined via $y = Pu \iff u = P^{-1}y$, with $P^{-1}(s) = \frac{1}{P(s)}$.

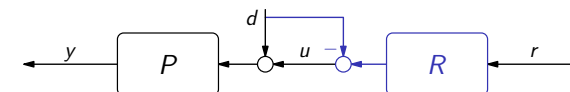
15/30

Open-loop plant inversion



with

$$R = P^{-1}$$

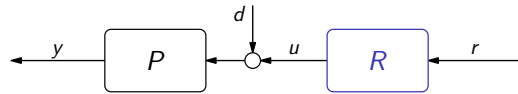


with

$$R = P^{-1}$$

16/30

Limitations of open-loop plant inversion: stability



All signals,

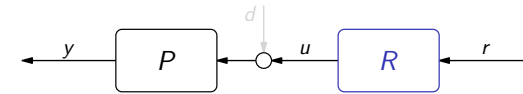
$$\begin{bmatrix} y \\ u \end{bmatrix} = \begin{bmatrix} PR & P \\ R & 0 \end{bmatrix} \begin{bmatrix} r \\ d \end{bmatrix},$$

bounded (internal stability).

Must have:

- P stable
- R stable, if $R = P^{-1} \implies P$ stably invertible

Approximate open-loop plant inversion



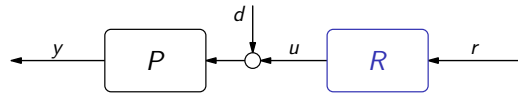
Pragmatic alternative:

$$R \approx P^{-1}r \quad \Rightarrow \quad R = P^{-1}T_{\text{ref}}r$$

Reference model:

- T_{ref} stable
- $P^{-1}T_{\text{ref}}$ stable (proper, poles in $\text{Re } s < 0$)
- $T_{\text{ref}} \approx 1$

Limitations of open-loop plant inversion: other



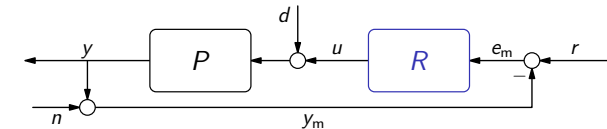
- unmeasured d
- uncertain P
- limited u

nothing to do

nothing to do

bandwidth limitations

Closed-loop control



Gang of four:

$$\begin{bmatrix} S(s) & T_c(s) \\ T_d(s) & T(s) \end{bmatrix} := \frac{1}{1 + P(s)R(s)} \begin{bmatrix} 1 & R(s) \\ P(s) & P(s)R(s) \end{bmatrix}$$

Signals:

$$\begin{bmatrix} y \\ u \\ e \end{bmatrix} = \begin{bmatrix} T & T_d & -T \\ T_c & -T & -T_c \\ S & -T_d & T \end{bmatrix} \begin{bmatrix} r \\ d \\ n \end{bmatrix},$$

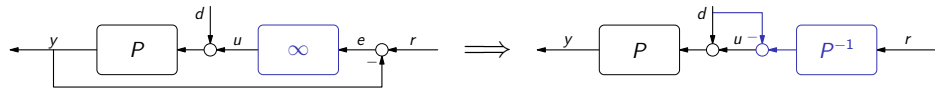
where $e := r - y = e_m + n$.

Closed-loop plant inversion

Because

$$T_c = \frac{1}{1/R + P} \xrightarrow{R \rightarrow \infty} \frac{1}{P} \quad \text{and} \quad -T = -\frac{P}{1/R + P} \xrightarrow{R \rightarrow \infty} -1,$$

we have



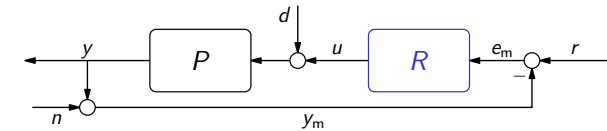
Thus,

$$T_d = \frac{P}{1 + PR} \xrightarrow{R \rightarrow \infty} 0 \quad \text{and} \quad S = \frac{1}{1 + PR} \xrightarrow{R \rightarrow \infty} 0,$$

independently of the plant and w/o explicit measurements of d .

21/30

Limitations of closed-loop plant inversion



- closed-loop stability
- closed-loop stability
- closed-loop stability
- measurement noise sensitivity
- limited u
- ...

Hence,

- nontrivial tradeoffs

22/30

Outline

Course info

Introduction

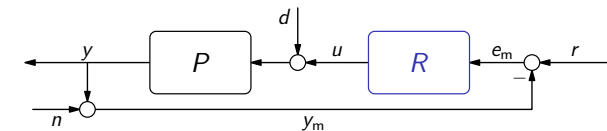
Review of signals and systems

Review of control principles

Naïve MIMO

23/30

Problem



for

$$P(s) = \begin{bmatrix} 1 + \alpha & 1 - \alpha \\ -1 + \alpha & -1 - \alpha \end{bmatrix}, \quad \alpha \in [0, 1]$$

Relations:

$$\begin{bmatrix} u \\ e \end{bmatrix} = \begin{bmatrix} T_c & -T_i \\ S_o & -T_d \end{bmatrix} \begin{bmatrix} r \\ d \end{bmatrix},$$

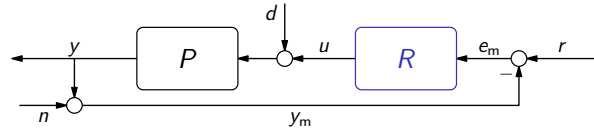
where

$$\begin{bmatrix} T_c(s) & T_i(s) \\ S_o(s) & T_d(s) \end{bmatrix} := \begin{bmatrix} R(s) \\ I \end{bmatrix} (I + P(s)R(s))^{-1} \begin{bmatrix} I & P(s) \end{bmatrix}.$$

($S_o \neq I - T_i$ in general).

24/30

Design 1



If

$$R(s) = k \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix},$$

then

$$\begin{bmatrix} T_i & T_c \end{bmatrix} = \frac{k}{2k+1} \begin{bmatrix} \frac{4\alpha k - 1 - \alpha}{2\alpha k + 1} & \frac{1 - \alpha}{2\alpha k + 1} & \vdots & \frac{(1 + \alpha)k + 1}{2\alpha k + 1} & \frac{(1 - \alpha)k}{2\alpha k + 1} \\ \frac{1 - \alpha}{2\alpha k + 1} & \frac{4\alpha k + 1 + \alpha}{2\alpha k + 1} & \vdots & -\frac{(1 - \alpha)k}{2\alpha k + 1} & -\frac{(1 + \alpha)k + 1}{2\alpha k + 1} \end{bmatrix}$$

and

$$\begin{bmatrix} T_d & S_o \end{bmatrix} = \frac{1}{2k+1} \begin{bmatrix} \frac{4\alpha k - 1 - \alpha}{2\alpha k + 1} & \frac{1 - \alpha}{2\alpha k + 1} & \vdots & \frac{(1 + \alpha)k + 1}{2\alpha k + 1} & \frac{(1 - \alpha)k}{2\alpha k + 1} \\ -\frac{1 - \alpha}{2\alpha k + 1} & -\frac{4\alpha k + 1 + \alpha}{2\alpha k + 1} & \vdots & \frac{(1 - \alpha)k}{2\alpha k + 1} & \frac{(1 + \alpha)k + 1}{2\alpha k + 1} \end{bmatrix}$$

25/30

Design 1 (contd)

If $\alpha \neq 0$, then

$$\lim_{k \rightarrow \infty} -T_i = -I \quad \text{and} \quad \lim_{k \rightarrow \infty} T_c = \frac{1}{4\alpha} \begin{bmatrix} 1 + \alpha & 1 - \alpha \\ -1 + \alpha & -1 - \alpha \end{bmatrix} = P^{-1}$$

and

$$u \rightarrow P^{-1}r - d \quad \text{and} \quad e \rightarrow 0,$$

exactly as in the SISO case.

26/30

Design 1 (contd)

If $\alpha = 0$ ($\det P = 0$), then

$$\lim_{k \rightarrow \infty} -T_i = -\frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \quad \text{and} \quad \lim_{k \rightarrow \infty} T_c = \frac{1}{2} \begin{bmatrix} k + 1 & k \\ -k & -k - 1 \end{bmatrix} \bigg|_{k \rightarrow \infty}$$

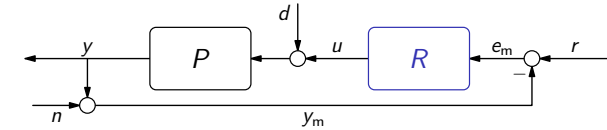
and

$$u \rightarrow \frac{1}{2} \begin{bmatrix} k + 1 & k \\ -k & -k - 1 \end{bmatrix} \bigg|_{k \rightarrow \infty} r - \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} d \quad \text{and} \quad e \rightarrow \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} r,$$

different from the SISO case.

27/30

Design 2



If

$$R(s) = k \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

then

$$\begin{bmatrix} T_i & T_c \end{bmatrix} = \frac{k}{4\alpha k^2 - 1} \begin{bmatrix} 4\alpha k - 1 - \alpha & -1 + \alpha & \vdots & (1 + \alpha)k - 1 & (1 - \alpha)k \\ 1 - \alpha & 4\alpha k + 1 + \alpha & \vdots & -(1 - \alpha)k & -(1 + \alpha)k - 1 \end{bmatrix}$$

and

$$\begin{bmatrix} T_d & S_o \end{bmatrix} = \frac{1}{4\alpha k^2 - 1} \begin{bmatrix} 4\alpha k - 1 - \alpha & -1 + \alpha & \vdots & (1 + \alpha)k - 1 & (1 - \alpha)k \\ 1 - \alpha & 4\alpha k + 1 + \alpha & \vdots & -(1 - \alpha)k & -(1 + \alpha)k - 1 \end{bmatrix}$$

28/30

Design 2 (contd)

If $\alpha \neq 0$, then

$$\lim_{k \rightarrow \infty} -T_i = -I \quad \text{and} \quad \lim_{k \rightarrow \infty} T_c = \frac{1}{4\alpha} \begin{bmatrix} 1+\alpha & 1-\alpha \\ -1+\alpha & -1-\alpha \end{bmatrix} = P^{-1}$$

and

$$u \rightarrow P^{-1}r - d \quad \text{and} \quad e \rightarrow 0,$$

exactly as in the SISO case.

Design 2 (contd)

If $\alpha = 0$ ($\det P = 0$), then

$$T_d = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}, \quad S_o = \begin{bmatrix} -k+1 & -k \\ k & k+1 \end{bmatrix}, \quad T_i = kT_d, \quad T_c = kS_o$$

and

$$u \rightarrow \left(\begin{bmatrix} -k(k-1) & -k^2 \\ k^2 & k(k+1) \end{bmatrix} r - k \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix} d \right) \Big|_{k \rightarrow \infty} \quad \text{and} \quad e \rightarrow \frac{1}{k}u$$

different from the SISO case.