Linear Control Systems (036012)

homework 5

(submission deadline: 16/4/2024, 20:00; do make an effort to be concise, clear, and accurate*)

Problem 1 (20pt)**.** Consider the following two generalized plants:

1. $G(s) = \left[\begin{array}{c:c} G_{11}(s) & G_{12}(s) \\ \hdashline G_{21}(s) & G_{22}(s) \end{array}\right] = \left[\begin{array}{c:c} 1/(s-2) & 1/(s-2) \\ 1 & 1 \\ \hdashline 1/(s-2) & 1/(s-2) \end{array}\right]$.

2. $G(s) = \left[\begin{array}{c:c} G_{11}(s) & G_{12}(s) \\ \hdashline G_{21}(s) & G_{22}(s) \end{array}\right] = \left[\begin{array}{c:c} 1/(s+2) & 1/(s-2) \\ 1 & 1 \\ \hdashline 1/(s-2) & 1/(s-2) \end{array}\right]$.

Are they internally stabilizable in the sense discussed in Sec. 6.4? If they are, parametrize all stabilizing controllers.

Problem 2 (20pt)**.** Let

$$P(s) = \left[\begin{array}{c|c} A & B \\ \hline I & 0 \end{array}\right],$$

which is a system whose whole state vector is measured, and assume that (A, B) is stabilizable.

1. Construct its doubly coprime factorization, in state space, in which the Bézout coefficients $X, Y, \tilde{X}, \tilde{Y}$ are static.

2. Parametrize all stabilizing controllers $K(s)$ for this plant in state space (in the spirit of (6.9)).

Problem 3 (30pt)**.** Consider a plant P having the transfer function

$$P(s) = \frac{2.25(s+1)(s-2)}{s(s^2-9)}.$$

1. Design all stabilizing LTI controllers for it.

2. Design all stabilizing LTI controllers *having an integral action* for it via imposing appropriate restrictions on the Youla parameter Q.

For both designs simulate the response of the resulted system (both the output and the control signal) to a step load disturbance and present those plots. Use controllers of the form of either one of the architectures in Fig. 6.4 in the Lecture Notes and use Simulink to simulate, with Q as a "transfer function" block. Use one Simulink file for both cases, with a manual switch between the unconstrained and constrained Youla parameters. In addition to the Simulink file, submit a stand-alone `*.m` file producing parameters for Simulink.

Problem 4 (30pt)**.** Let P be a plant with the transfer function $P(s) = 1/(s-1)$. The goal is to stabilize it with minimum control effort, measured by a size of the control sensitivity transfer function (mind the use of positive feedback) $T_c(s) = R(s)/(1 - P(s)R(s))$.

1. What is the smallest attainable $\|T_c\|_\infty$? What controller $R(s)$ attains it?

2. Assume that the bound $|T_c(j\omega)| \le 1$ has to be met for all $\omega > \omega_0$ for some $\omega_0 > 0$. What is the lower bound on $\|T_c\|_\infty$ in this case ? Plot this bound as a function of ω_0.

*Computations may be done with whatever numerical or symbolic tools. But the logic should be explained and proofs given.