הטכניון – מכון טכנולוגי לישראל, הפקולטה להנדסת מכונות
TECHNION—Israel Institute of Technology, Faculty of Mechanical Engineering

Linear Control Systems (036012)

homework 3

(submission deadline: 12/3/2024, 20:00; do make an effort to be concise, clear, and accurate*)

Problem 1 (25%)**.** Let $G_1(s) = \left[\begin{array}{c|c} A & B \\ \hline C_1 & D_1 \end{array}\right]$ and $G_2(s) = \left[\begin{array}{c|c} A & B \\ \hline C_2 & D_2 \end{array}\right]$ be minimal realizations. Prove that

$$G_{\text{stack}}(s) := \begin{bmatrix} G_1(s) \\ G_2(s) \end{bmatrix} = \left[\begin{array}{c|c} A & B \\ \hline C_1 & D_1 \\ C_2 & D_2 \end{array}\right]$$

and that this realization is minimal as well.

Problem 2 (25pt)**.** Prove that (C, A) is observable iff there is no vector $\eta \neq 0$ such that $C(sI - A)^{-1}\eta \equiv 0$.

Problem 3 (50pt)**.** Consider the following transfer matrices:

$$G_2(s) = \begin{bmatrix} -\frac{2s-9}{s-2} & -1 \\ -\frac{7}{s-2} & 1 \end{bmatrix} \quad \text{and} \quad G_1(s) = \begin{bmatrix} \frac{s^2+2s-1}{s^2-1} & \frac{s^2+1}{s^2-1} \\ \frac{-s-5}{2s^2-2} & \frac{2s^2-5s-3}{2s^2-2} \end{bmatrix}.$$

1. Construct their Gilbert's state-space realizations and calculate corresponding realization poles, invariant zeros, and their directions using the state-space formulae.
2. Construct the state-space realization of $G_2(s)G_1(s)$ using the formula from §4.1.1. Is it minimal? Explain why.
3. If the realization in the previous item is not minimal, find its hidden modes and construct a minimal realization of the system.

*Computations may be done with whatever numerical or symbolic tools. But the logic should be explained and proofs given.