הטכניון – מכון טכנולוגי לישראל, הפקולטה להנדסת מכונות
TECHNION—Israel Institute of Technology, Faculty of Mechanical Engineering

Linear Control Systems (036012)

HOMEWORK 2

(submission deadline: 27/2/2024, 20:00; do make an effort to be concise, clear, and accurate*)

Problem 1 (40pt)**.** Calculate the H_∞ and H_2 norms of the following transfer functions:

1. $G_1(s) = k\frac{\tau_1 s-1}{\tau_1 s+1}$,
2. $G_2(s) = k\frac{-\tau_1 s+1}{(\tau_1 s+1)(\tau_2 s+1)}$,
3. $G_3(s) = k\frac{\tau_1 s+1}{(-\tau_1 s+1)(\tau_2 s+1)(\tau_3 s+1)}$,
4. $G_4(s) = \frac{1-2\mathrm{e}^{-s}}{(\tau_1 s+1)(2-\mathrm{e}^{-s})}$,

assuming that $k > 0$ and $\tau_i > 0$ for all $i \in \mathbb{Z}_{1..3}$.

Problem 2 (20pt)**.** Are the pairs below coprime in H_∞:

1. $M(s) = \frac{s+1}{s-1}$ and $N(s) = \frac{1}{s-1}$?
2. $M(s) = \frac{s-1}{s+1}$ and $N(s) = \frac{1}{s+1}$?

If the answer is "yes," find the corresponding Bézout coefficients.

Problem 3 (40pt)**.** Consider the transfer matrices

$$G_1(s) = \begin{bmatrix} \frac{s^2+2s-1}{s^2-1} & \frac{s^2+1}{s^2-1} \\ \frac{-s-5}{2s^2-2} & \frac{2s^2-5s-3}{2s^2-2} \end{bmatrix} \quad \text{and} \quad G_2(s) = \begin{bmatrix} -\frac{2s-9}{s-2} & -1 \\ -\frac{7}{s-2} & 1 \end{bmatrix}.$$

1. Find McMillan degree, poles, zeros, and directions (input and output) of poles and zeros of $G_1(s)$.
2. Find McMillan degree, poles, zeros, and directions (input and output) of poles and zeros of $G_2(s)$.
3. Find McMillan degrees and poles of $G_1(s)G_2(s)$ and $G_2(s)G_1(s)$. Explain differences.

Problem 4 (bonus, 15pt)**.** Is the transfer function $G(s) = 1/(1 + s\mathrm{e}^{-s})$ proper ?

*Computations may be done with whatever numerical or symbolic tools. But the logic should be explained and proofs given.