הטכניון – מכון טכנולוגי לישראל, הפקולטה להנדסת מכונות
TECHNION—Israel Institute of Technology, Faculty of Mechanical Engineering

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דוגמאות בחינות סופיות

Tammuz 21, 5783 July 10, 2023

Part I

Frequency-domain methods

Question 1

Suggest a first-order system, whose addition to the loop gain adds a phase lag of 170° at certain frequency.

$G(s) =$ ______________, because ____

____

Question 2

Fig 1(a) presents the Nichols chart of an open-loop transfer function $L(s)$. What is the *tightest* estimate of the closed-loop bandwidth, which can be deduced from this plot?

$\omega_b \in$ __________________, because ____

____

Question 3

Fig 1(b) presents the Nichols chart of an open-loop transfer function $L(s)$. What is the *tightest* estimate of the closed-loop bandwidth, which can be deduced from this plot?

$\omega_b \in$ __________________, because ____

____

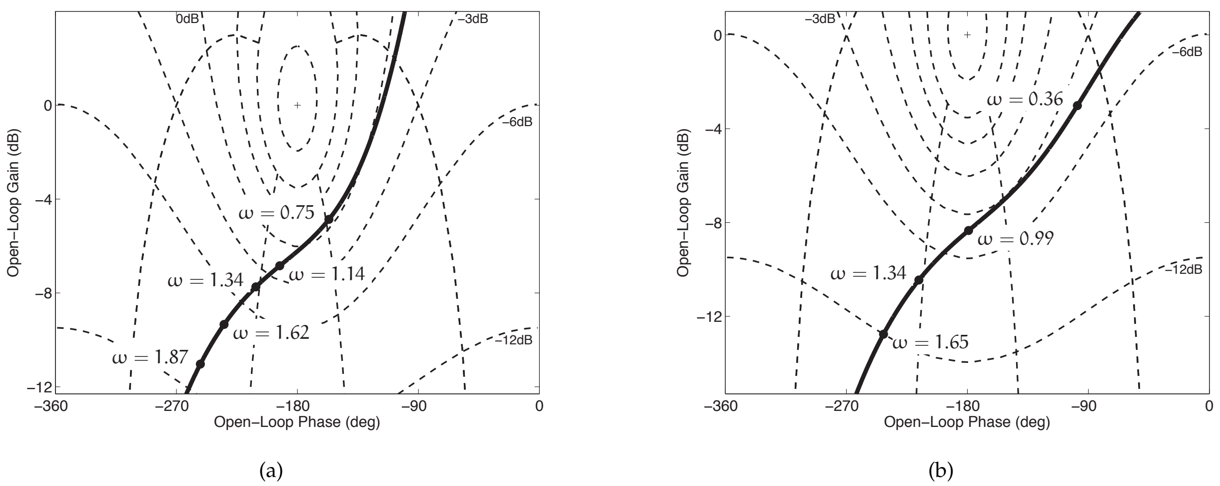
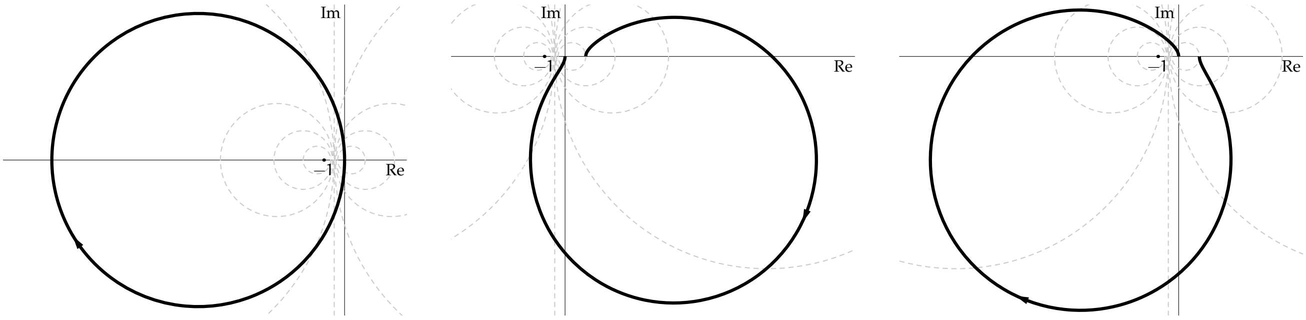
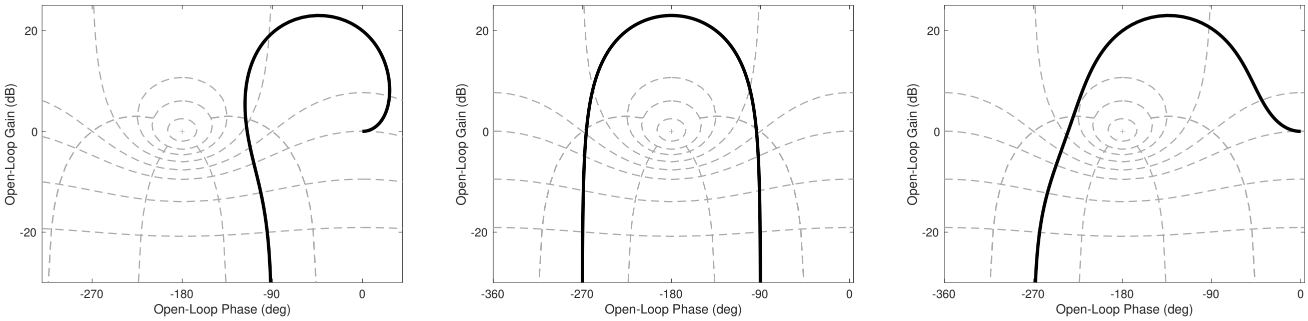


Figure 1: Nichols charts



(a) $P_1(s)$ (b) $P_2(s)$ (c) $P_3(s)$

Figure 2: polar plots



(a) $P_\alpha(s)$ (b) $P_\beta(s)$ (c) $P_\gamma(s)$

Figure 3: Nichols charts

Question 4

Figs. 2 and 3 present polar plots and Nichols charts, respectively, of 3 stable systems. Find correspondences between these plots.

$P_\alpha = P$__, because ____________________________

$P_\beta = P$__, because ____________________________

$P_\gamma = P$__, because ____________________________

Question 5

For each one of the systems in Fig. 3, determine if there is output feedback $u = -ky$ with $|k| \geqslant 1$, for which the closed-loop system is stable. If the answer is affirmative, then provide an example of such feedback law.

stabilizing $|k| \geqslant 1$ for P_α does / doesn't exist, because ____________________

____

stabilizing $|k| \geqslant 1$ for P_β does / doesn't exist, because ____________________

____

stabilizing $|k| \geqslant 1$ for P_γ does / doesn't exist, because ____________________

____

Question 6

What is the magnitude of the frequency response of the 2-order Butterworth filter with the bandwidth 2 $\frac{\text{rad}}{\text{sec}}$?

$|F(j\omega)| =$ ____________________

Question 7

Let $L(s) = \dfrac{b_1 s + 1}{s^2 + s + 1}$ for some $b_1 \geqslant 0$. For what b_1 we have $\displaystyle\int_0^\infty \ln|S(j\omega)|d\omega = 0$, where $S(s) = 1/(1 + L(s))$?

$b_1 =$ ________, because ____________________

Question 8

Given $P(s) = s(s-1)/(s^3 + s + 1)$, what is the minimum-degree closed-loop characteristic polynomial $\chi_{cl}(s)$ for which all closed-loop poles are in $s = -1$ for the pole-placement approach based on the Sylvester matrix ?

$\chi_{cl}(s) =$ ____________, because ____________________

____

Question 9

Given $P(s) = s(s-1)/(s^3 + s + 1)$, what is the minimum-degree closed-loop characteristic polynomial $\chi_{cl}(s)$ for which all closed-loop poles are in $s = -1$ and the controller includes an integral action for the pole-placement approach based on the Sylvester matrix ?

$\chi_{cl}(s) =$ ____________, because ____________________

____

Question 10

Is the system $P(s) = (s-10)^3/(s+1)^5$ strongly stabilizable ?

yes / no, because ____________________

Question 11

Can $R(s) = \dfrac{-s^5 + s^4 + s^3 + s^2 - s + 1}{s^5 + s^4 - s^3 + s^2 + s + 1}$ be a Padé approximant of e^{-sh} for some $h > 0$?

yes / no, because ____________________

Question 12

Can $R(s) = \dfrac{-s^3 + 6s^2 - 6s + 1}{s^3 + 3s^2 + 3s + 1}$ be a Padé approximant of e^{-sh} for some $h > 0$?

yes / no, because ____________________

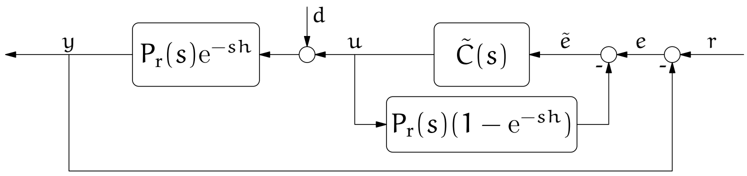


Figure 4: Dead-time system

Question 13

What is the complementary sensitivity function (from r to y) for the system in Fig. 4, where $P_r(s) = \frac{s+8}{s(s+4)}$, $\tilde{C}(s) = \frac{s+4}{s+8}$, and $h = 0.1$?

$T(s) =$ ______________________________________

Question 14

Is the system in Fig. 4 internally stable under $P_r(s) = \frac{(s+3)^2+4}{(s-1)(s+3)}$, $\tilde{C}(s) = 10$, and $h = 0.2$?

yes / no, because ______________________________________

Question 15

Is the system in Fig. 4 internally stable under $P_r(s) = \frac{7}{s(s+3)^2}$, $\tilde{C}(s) = 10$, and $h = 0.2$?

yes / no, because ______________________________________

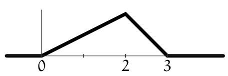
Question 16

Which one of the transfer functions below does not correspond to an FIR system:

$$G_1(s) = \frac{1-e^{-2\pi s}}{s^2}, \quad G_2(s) = \frac{1-e^{-2\pi s}}{s^2+1}, \quad \text{and} \quad G_3(s) = \frac{1-2\pi s - e^{-2\pi s}}{s^2} \,?$$

$G_{\_}(s)$, because ______________________________________

Question 17

For what α_1 and α_2 the impulse response of $G(s) = \dfrac{1+\alpha_1 e^{-2s} + \alpha_2 e^{-3s}}{2s^2}$ is $g(t) =$  ?

$\alpha_1 =$ ________ and $\alpha_2 =$ ________, because ______________________________

Question 18

Given $P(s) = 1/s$, what is the control signal $u(t)$, which causes the output to move from $y(0) = 0$ to $y(t_f) = 10$ in minimum time under $|u(t)| \leqslant 1$?

$u(t) =$ ____________________, because ____

Question 19

Given $P(s) = (s-1)/(s^2+3s+2)$. Is there a stabilizing controller such that the resulting complementary sensitivity function is $T(s) = 4/((s+30)(s^2+2\sqrt{2}s+4))$? If there is, what is it ?

yes / no, because ____

Question 20

Suppose that the open-loop transfer function $L(s)$ is stable and satisfis the relation

$$\left|\frac{L(j\omega)}{L_0(j\omega)} - 1\right| \leqslant \frac{|\omega|}{3}, \quad \forall\omega$$

for a known $L_0(s)$. Under what conditions the closed-loop system is stable for all such $L(s)$? (no need to explain)

____

Question 21

Given a plant $P(s)$ belonging to the uncertainty set $\mathfrak{P}_\omega = \left\{P(j\omega) : \left|\frac{P(j\omega)}{P_0(j\omega)} - 1\right| \leqslant \ell_P(\omega)\right\}$ for the nominal plant $P_0(s) = 1/(s(s+1)^2)$ and uncertainty radius $\ell_P(\omega) \geqslant 0$. Is there a stabilizing controller $C(s)$, for which the complementary sensitivity function $T(s) = P(s)C(s)/(1+P(s)C(s))$ belongs to the uncertainty set

$$\mathfrak{T}_\omega = \left\{T(j\omega) : \left|\frac{T(j\omega)}{T_0(j\omega)} - 1\right| \leqslant \ell_T(\omega)\right\}$$

for $T_0(s) = P_0(s)C(s)/(1+P_0(s)C(s))$ and $\ell_T(\omega) \leqslant \ell_P(\omega)$ for all ω ?

yes / no, because ____

____

Question 22

Fig. 5 presents a control system with the controller $C(s) = \frac{1}{s-1}$ and an anti-windup mechanism. What is the admissible range of the gain k_{awu} ?

$k_{awu} \in$ __________________, because ____

____

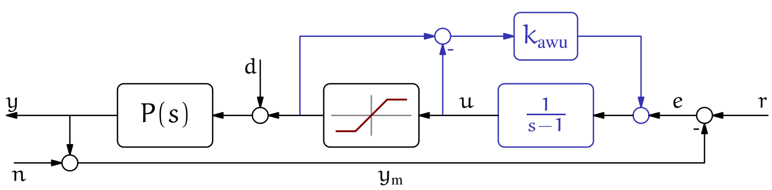


Figure 5: Control system with anti-windup

Part II

State space

Question 23

Is $(A - I)^4 = 0$ for $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 5 & 1 \end{bmatrix}$?

yes / no, because ____

Question 24

Could it be true that $\exp\left(\begin{bmatrix} -1 & 1 \\ 2 & 2 \end{bmatrix} t\right) = \begin{bmatrix} e^{-t} & e^{2t} - e^{-t} \\ 0 & e^{2t} \end{bmatrix}$?

yes / no, because ____

Question 25

Calculate $\Phi(t) = \exp\left(\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} t\right)$. No need to explain.

$$\Phi(t) = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$$

Question 26

Is the matrix $M = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} > 0$?

yes / no, because ____

Question 27

Is the matrix $M = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} > 0$?

yes / no, because ____

Question 28

What is the pole excess of $G(s)$, whose state-space realization is $G : \begin{cases} \dot{x}(t) = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u(t) \\ y(t) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x(t) \end{cases}$?

$n - m =$ ______, because ____

Question 29

Given $G : \begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases}$. Derive a state-space realization of $2G + 1$ whose dimension matches that of G.

Question 30

Given $G : \begin{cases} \dot{x} = Ax + Bu \\ y = Cx + 2u \end{cases}$. Derive a state-space realization of $(G - 1)^{-1}$ whose dimension matches that of G.

Question 31

Is the system $\dot{x}_1 = x_2,\ \dot{x}_2 = u$ stable ?

yes / no, because ____________________

Question 32

Consider a second-order system of the form $\dot{x}(t) = Ax(t)+Bu(t)$ and assume that there is a control law attaining $x(5) = \left[\begin{smallmatrix}1\\1\end{smallmatrix}\right]$ from all $x(0)$. Is the system controllable ?

yes / no, because ____________________

Question 33

Consider a second-order system of the form $\dot{x}(t) = Ax(t)+Bu(t)$ and assume that there is a control law attaining $x(5) = \left[\begin{smallmatrix}1\\1\end{smallmatrix}\right]$ from $x(0) = 0$. Is the system controllable ?

yes / no, because ____________________

Question 34

Is the system $\dot{x}(t) = \left[\begin{smallmatrix}2&3\\0&1\end{smallmatrix}\right] x(t) + \left[\begin{smallmatrix}0\\1\end{smallmatrix}\right] u(t)$ controllable ?

yes / no, because ____________________

Question 35

Let $X = \begin{bmatrix}1&1\\1&0\end{bmatrix}$ and $Y = \begin{bmatrix}1\\1\end{bmatrix}$. Is the matrix $\int_0^{0.00001} e^{Xt}YY'e^{X't}dt$ singular (i.e. not invertible) ?

singular / nonsingular, because ____________________

Question 36

What mode of $\left(\left[\begin{smallmatrix}1&1\\0&2\end{smallmatrix}\right], \left[\begin{smallmatrix}1\\1\end{smallmatrix}\right]\right)$ is uncontrollable ?

$\lambda =$ ____, because ____________________

Question 37

Is the realization with $A = \left[\begin{smallmatrix}0&0\\0&1\end{smallmatrix}\right]$, $B = \left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]$ and $C = \left[\begin{smallmatrix}1&1\end{smallmatrix}\right]$ minimal ?

yes / no, because ____________________

Question 38

Find a minimal realization for the system $\begin{cases}\dot{x} = \left[\begin{smallmatrix}3&0\\0&8\end{smallmatrix}\right]x + \left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]u\\ y = \left[\begin{smallmatrix}0&1\end{smallmatrix}\right]x\end{cases}$.

____

Question 39

Is the system $\dot{x} = \begin{bmatrix} -1 & 4.5 & 1 & 9 \\ 0 & 2 & 0 & -1 \\ 0 & 0 & -0.5 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} u$ stabilizable ?

yes / no, because ______________________________

Question 40

Design the state-feedback gain assigning all closed-loop eigenvalues at -1 for the system

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$

$K =$ ______________, because ______________________________

____

Question 41

Can $T_{yr}(s) = 1/(s+1)^3$ be the closed-loop transfer function for a stabilizing state feedback for $P(s) = (s+2)/s^4$?

yes / no, because ______________________________

Question 42

Can $T_{yr}(s) = 1/(s+2)^4$ be the closed-loop transfer function for a stabilizing state feedback for $P(s) = (s+2)/s^4$?

yes / no, because ______________________________

Question 43

The standard Luenberger observer for the system

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t), & x(0) = x_0, \\ y(t) = Cx(t) \end{cases}$$

is

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + L(y(t) - C\hat{x}(t)), \quad \hat{x}(0) = 0.$$

It is known to render the estimation error $\epsilon(t) \doteq x(t) - \hat{x}(t)$ independent of the control signal $u(t)$ and the convergence of the error can be affected by the choice of the gain L, provided (C, A) is observable. Suggest an observer to the system

$$\begin{cases} \dot{x}(t) = Ax(t), & x(0) = x_0, \\ y(t) = Cx(t) + Du(t) \end{cases}$$

with the same properties (estimation error is independent of $u(t)$ and its convergence can be affected by a design parameter). Prove that via presenting the dynamic equation for $\epsilon(t)$.

Question 44

Is $\bar{X} = \left[\begin{smallmatrix} 1 & 0 \\ 0 & -1 \end{smallmatrix}\right]$ the stabilizing solution to the Riccati equation

$$\left[\begin{smallmatrix} 0 & 0 \\ 1 & 0 \end{smallmatrix}\right]\bar{X} + \bar{X}\left[\begin{smallmatrix} 0 & 1 \\ 0 & 0 \end{smallmatrix}\right] + \left[\begin{smallmatrix} 1 & -1 \\ -1 & 1 \end{smallmatrix}\right] - \bar{X}\left[\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}\right]\bar{X} = 0\,?$$

yes / no, because ______________________________

Question 45

Is $\bar{X} = \left[\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}\right]$ the stabilizing solution to the Riccati equation

$$\left[\begin{smallmatrix} 0 & 0 \\ 1 & 0 \end{smallmatrix}\right]\bar{X} + \bar{X}\left[\begin{smallmatrix} 0 & 1 \\ 0 & 0 \end{smallmatrix}\right] + \tfrac{1}{2}\left[\begin{smallmatrix} 1 & -1 \\ -1 & 1 \end{smallmatrix}\right] - \tfrac{1}{2}\bar{X}\left[\begin{smallmatrix} 1 & 1 \\ 1 & 1 \end{smallmatrix}\right]\bar{X} = 0\,?$$

yes / no, because ______________________________

____

Question 46

Is there a similarity transformation between the realizations

$$\begin{cases} \dot{x}_2(t) = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} x_2(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \\ y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x_2(t) \end{cases} \quad \text{and} \quad \begin{cases} \dot{x}_1(t) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} x_1(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \\ y(t) = \begin{bmatrix} 1 & 1 \end{bmatrix} x_1(t) \end{cases} ?$$

If yes, find its transformation matrix T.

yes / no, because ______________________________

Question 47

Is there a similarity transformation between the realizations

$$\begin{cases} \dot{x}_2(t) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} x_2(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \\ y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x_2(t) \end{cases} \quad \text{and} \quad \begin{cases} \dot{x}_1(t) = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} x_1(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \\ y(t) = \begin{bmatrix} 1 & 1 \end{bmatrix} x_1(t) \end{cases} ?$$

If yes, find its transformation matrix T.

yes / no, because ______________________________

Question 48

The loop transfer function for a state feedback, $u = Kx$, designed by LQR with $S = 0$ is $L(s) = -K(sI - A)^{-1}B$. Can $L(s) = 1/(s+1)^2$ be such a transfer function ?

yes / no, because ______________________________

Question 49

The loop transfer function for a state feedback, $u = Kx$, designed by LQR with $S = 0$ is $L(s) = -K(sI - A)^{-1}B$. Can $L(s) = (s-1)/(s+1)^2$ be such a transfer function ?

yes / no, because ______________________________

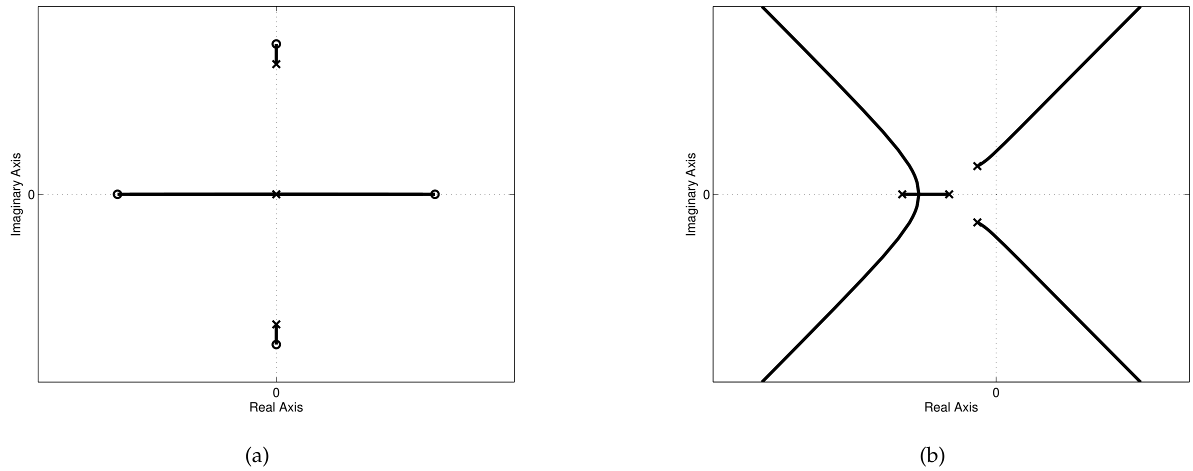


Figure 6: Root-locus plots

Question 50

Is a third-order state-space realization of $L(s) = P(s)C(s)$, where $P(s) = 1/(s^2+2s+1)$ and $C(s) = (2s+2)/(s+4)$ (a lead controller), minimal ?

yes / no, because ____

Question 51

Can the root locus in Fig. 6(a) belong to $1 + \frac{1}{r}P(-s)QP(s) = 0$ under $r > 0$ and $Q \geqslant 0$?

yes / no, because ____

Question 52

Can the root locus in Fig. 6(b) belong to $1 + \frac{1}{r}P(-s)QP(s) = 0$ under $r > 0$ and $Q \geqslant 0$?

yes / no, because ____

Question 53

Consider the plant $P(s) = 1/(s^2 - a^2)$ for $a < 0$. Write its state-space realization and design the state feedback law $u(t) = Kx(t)$ minimizing $\int_0^\infty u^2(t)dt$.

Question 54

Write a quadratic cost function, whose minimization for an LTI plant with a measured state guarantees that closed-loop poles are in $\{s \mid \operatorname{Re} s < -2\}$.

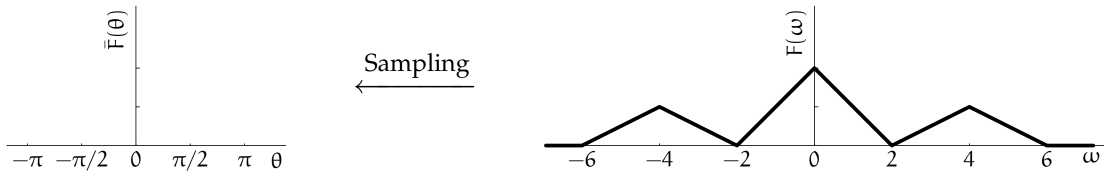
Question 55

The return-difference equality for the LQR problem is $1 + \frac{1}{r}B'(-j\omega I - A')^{-1}Q(j\omega I - A)^{-1}B = |1 + L(j\omega)|^2$, where $L(s) = -K(sI - A)^{-1}B$ is the loop transfer function. Prove that the LQR controller guarantees a phase margin of at least 60°.

Part III
Sampled-data systems

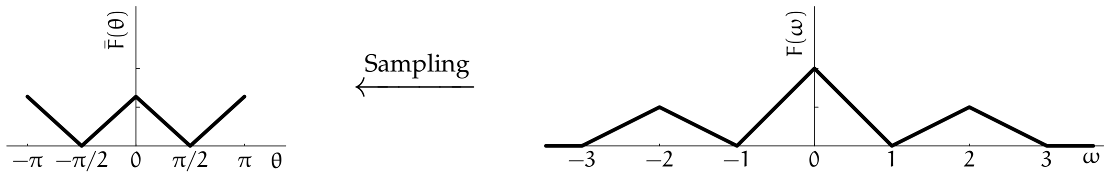
Question 56

The figure below presents the spectrum of an analog signal $f(t)$. Draw the spectrum of its sampled version $\bar{f}[i] = f(ih)$ under the sampling period $h = \frac{\pi}{4}$ (assume that the axes of $F(\omega)$ and $\bar{F}(\theta)$ are compatible).



Question 57

The spectra of an analog signal $f(t)$ and its sampled version $\bar{f}[i] = f(ih)$ are presented below. What is the sampling period h?



Question 58

Frequency range of an adult human ear is $20 \div 20,000$ Hz. Choose the bandwidth of the ideal anti-aliasing filter to be placed in a microphone before converting a record to a digital form. Explain briefly.

$\omega_b =$ ________, because ____

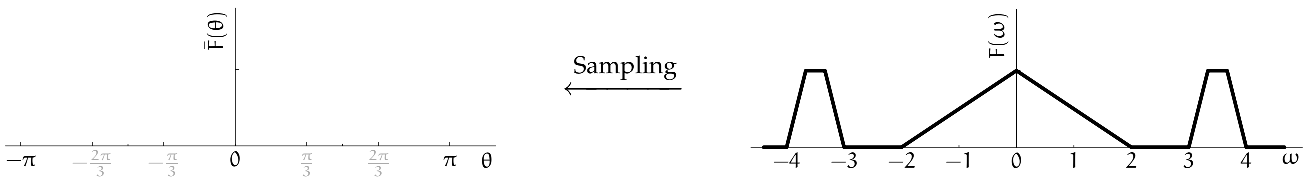


Figure 7: Spectrum of $f(t)$

Question 59

Fig. 7 depicts the spectrum of an analog signal $f(t)$. What is the maximal sampling period h for which $f(t)$ can be perfectly reconstructed from its sampling $\bar{f}[i] = f(ih)$? Draw the spectrum of $\bar{f}[i]$ for the chosen h.

$h_{max} =$ ________, because ____

____

Question 60

Can $\bar{C}(z) = (z^2+z+1)/(z^2+4z+4)$ be the Tustin approximant of $C(s) = (s+1)/(s^2+3s+1)$ for some sampling period?

yes / no, because ____

Question 61

Can $\bar{C}(z) = (z^2 + z + 1)/(z - 1)^2$ be the Tustin approximant of $C(s) = (s + 1)/s^2$ for some sampling period ?

yes / no, because ______________________________

Question 62

Can $\bar{C}(z) = (z + 1)^4/(3z - 1)^4$ be the Tustin approximant of $C(s) = (s + 1)^3/(s + 2)^4$ for some sampling period ?

yes / no, because ______________________________

Question 63

Can $\bar{P}(z) = (z + 1)^4/(3z - 1)^4$ be the discretization of $P(s) = (s + 2)^3/(s + 1)^4$ for some sampling period ?

yes / no, because ______________________________

Question 64

Can $\bar{P}(z) = (z + 1)^3/(z - 1)^4$ be the discretization of $P(s) = (s + 2)^3/(s + 1)^4$ for some sampling period ?

yes / no, because ______________________________

Question 65

Under what conditions on a the system $x[t + 1] = \frac{4}{3+4a^2}x[t] + \pi^8 u[t]$ is stable ?

$|a| \in$ ____________, because ______________________________

____

Question 66

Is there a sampling period h for which the discrete system below is the discretization of the analog system below under $\bar{x}[k] = x(kh)$?

$$\dot{x}(t) = \begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t) \qquad \longrightarrow \qquad \bar{x}[i + 1] = \begin{bmatrix} 0.5 & 0.3 \\ 0 & 0.25 \end{bmatrix} \bar{x}[i] + \begin{bmatrix} 1.72 \\ 0 \end{bmatrix} \bar{u}[i]$$

yes / no, because ______________________________

Question 67

What sampling period h is pathological for a system with the transfer function $G(s) = s/(s^2 - 1)$?

$h =$ ________, because ______________________________

____

Formulae[1]

- Constants: $e = 2.7182818284590452354$, $\pi = 3.1415926535897932385$
- Power series: $F(s) = F(0) + \frac{F'(0)}{1!}s + \frac{F''(0)}{2!}s^2 + \frac{F'''(0)}{3!}s^3 + \cdots$
- Bode's sensitivity integral (provided the pole excess of $L(s)$ is at least 2):

$$\int_{-\infty}^{\infty} \ln|S(j\omega)| = \begin{cases} 0 & \text{if } L(s) \text{ is stable} \\ \pi \sum_i \operatorname{Re} p_i & \text{otherwise} \end{cases}$$

- Ackermann's formula: $F = \begin{bmatrix} 0 & \cdots & 0 & 1 \end{bmatrix} M_c^{-1} \chi_{cl}(A)$.
- Continuous-time optimal LQR control law $u(t) = -R^{-1}(S' + B'\bar{X})x(t)$, where $\bar{X} = \bar{X}' \geqslant 0$ is the stabilizing solution to the ARE

$$A'\bar{X} + \bar{X}A + Q - (S + \bar{X}B)R^{-1}(S' + B'\bar{X}) = 0, \quad \text{where } R > 0 \text{ and } \begin{bmatrix} Q & S \\ S' & R \end{bmatrix} \geqslant 0$$

- Discretization:

$$\dot{x}(t) = Ax(t) + Bu(t) \quad \rightarrow \quad \bar{x}[i+1] = e^{Ah}\bar{x}[i] + \int_0^h e^{At}dtB\,\bar{u}[i]$$

[1]Don't pavlov on their use.