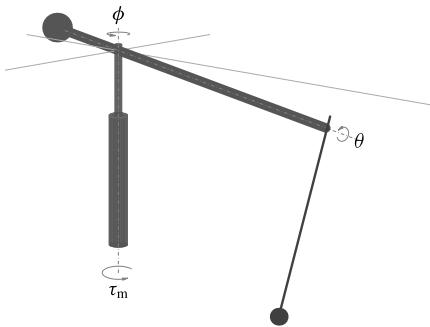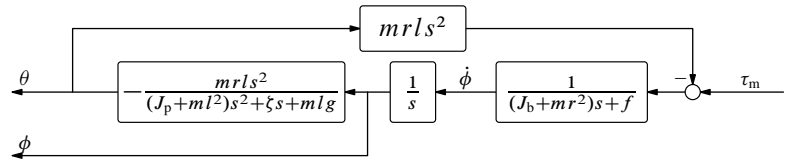# תורת הבקרה (00350188)

סמסטר אביב תשפ"ה

פרויקט מס' 1

(a) Schematic diagram

(b) Block-diagram of its linearized model

Figure 1: Furuta pendulum

1 Setup

The Furuta pendulum consists of a driven arm, which rotates in the horizontal plane, and a pendulum attached to it, which rotates in the vertical plane. The arm is mounted on a vertical shaft as shown in Fig. 1(a). The whole system is rotated by applying a torque τ_{m} to the shaft. The arm and pendulum angles are denoted as ϕ and θ, respectively (the shaft angle equals that of the arm). The control goal her is twofold:

 (i) to control the arm position (angle), so it could follow a reference setpoint trajectory ϕ_{ref},

 (ii) to ensure that the pendulum does not oscillate "too much" during transients of the arm.

The torque applied to the shaft is generated by a DC motor, so we assume that the rod includes the motor rotor and gearhed as its part.

2 Modeling

The block diagram of a linearized model of the system in Fig. 1(a) around its equilibrium at $\theta = 0$ rad with the torque τ_{m} as the input and the angles ϕ and θ (in rad) as the outputs is presented in Fig. 1(b), where

 • J_{b} and f are the total moment of inertia (around its pivot point) and viscous friction of the rotating base (rod + arm);

 • r is the distance from the pendulum rotation plane to the arm pivot point;

 • m is the pendulum mass;

 • l is the distance from the pendulum pivot point to its centroid;

 • J_{p} and ζ are the moment of inertia (around its centroid) and damping coefficient of the pendulum, respectively;

 • g is the standard gravity.

Let τ_{m} be generated by a DC motor having the torque and back emf constants K_{m} and $K_{\mathrm{b}} = K_{\mathrm{m}}$, respectively, the resistance R, a negligible inductance, and a gearbox with the ratio N_{g} (so that the angular veocity of the motor shaft is $N_{\mathrm{g}}\dot{\phi}$ and $\tau_{\mathrm{m}} = N_{\mathrm{g}}\tau_0$, where τ_0 is the torque generated by the rotor). The armature voltage of the motor is generated by an amplifier, modeled as a static gain K_{a}.

Parameter	l	r	m	J_{p}	ζ	J_{b}	f	g	R	K_{m}	N_{g}	K_{a}
Value (in SI)	0.3226	0.3	0.04	0.0003	0.0005	0.01	0.01	9.81	2.08	0.0243	4.5	12

Table 1: Parameters of the system

1. Present the block-diagram of the overall system, having the amplifier input signal, u, as its input and θ and ϕ (in rad) as its outputs.

2. Derive the transfer functions of $P_{\phi u} : u \mapsto \phi$, $P_{\theta u} : u \mapsto \theta$, and $P_{\theta \phi} : \phi \mapsto \theta$ in symbolic form. Also derive the transfer function of $P_{\phi u 1} := \lim_{\zeta \to \infty} P_{\phi u}$, which corresponds to the system with a locked pendulum.

3. Assuming that the system parameters are as given in Table 1, present the Bode diagrams of the frequency responses of the systems from the previous item (the Bode plots of $P_{\phi u}$ and $P_{\phi u 1}$ should be presented in the same figure). Make sure that the resonance of $P_{\phi u}$ is at ≈ 6.1 rad/sec.

3   Servo loop for positioning ϕ

Our first goal is to design a servo loop for controlling the arm position ϕ. Apart from controlling ϕ itself, this loop has an additional goal of reducing the sensitivity of the arm position to pendulum oscillations. To this end, the bandwidth of the servo loop should substantially exceed the resonance frequency of the pendulum.

4. Design a minimal-order feedback controller C in the setup shown in Fig. 2(a) (ignoring $P_{\theta \phi}$) such that

    (a) steady-state responses / errors to step d and ϕ_{ref} vanish,

    (b) the crossover frequency of this loop $\omega_{\mathrm{c}} \approx 60$ rad/sec,

    (c) the phase margin $\mu_{\mathrm{ph}} \approx 45°$,

    (d) the magnitude of the complementary sensitivity frequency response, $T_{\phi}(\mathrm{j}\omega)$, does not exceed $+3$ dB,

    (e) the magnitude of the disturbance sensitivity frequency response, $T_{\phi,\mathrm{d}}(\mathrm{j}\omega)$, does not exceed -20 dB

    (the meaning of "$\approx$" is that at most 5% deviations are allowed; consider designing the lead part of the controller so that its maximum phase lead is at a frequency slightly *below* the required crossover and use M-circles to tune them). Present the Bode magnitude plots of the resulted complementary and disturbance sensitivity functions and determine the resulted closed-loop bandwidth. Plot also time responses of the closed-loop system (namely, the signals ϕ, u, and θ) to a load disturbance $d = 4/(s + 4)\mathbb{1}$, where $\mathbb{1}$ is the unit step.

4   Choice of setpoint tracking signals ϕ_{ref} and u_{req}

Now we can concentrate on tracking requirements. We aim at moving the arm from its initial equilibrium $\phi(0) = \phi_0$ to its final equilibrium $\phi(t_{\mathrm{f}}) = \phi_{\mathrm{f}}$ in minimum time t_{f}. Two additional requirements are of importance in choosing the control strategy for that. First, we need to do this under realistic control effort. In this project this is expressed in the requirement to keep the torque generated by the motor (after the gearbox) bounded, in the sense $|\tau_{\mathrm{m}}(t)| \leq \tau_{\mathrm{max}}$ for some $\tau_{\mathrm{max}} > 0$. Second, we would like to arrive at the destination with "pacified" pendulum, i.e. with $\theta(t_{\mathrm{f}}) = \dot{\theta}(t_{\mathrm{f}}) = 0$. This problem could, in principle, be solved as the time-optimal optimization problem. However, such a problem would be technically challenging and require methods, whose mastering goes far beyond the scope of this course. We shall therefore follow a pragmatic alternative, in which the arm reference and the required control input are designed in two steps.
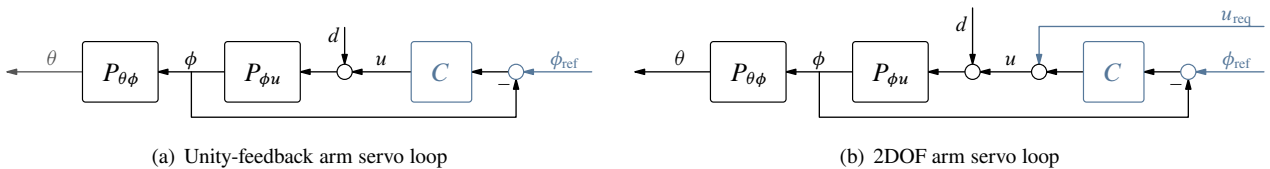

(a) Unity-feedback arm servo loop   (b) 2DOF arm servo loop

Figure 2: 2DOF controller for the pendulum

5. Consider first the "stripped down" system $P_{\phi u1}$, in which the pendulum is mounted but locked (i.e. it cannot oscillate). Derive the transfer function from the motor torque τ_m to the arm angle ϕ there. Design the optimal torque τ_m such that the angle ϕ reaches its final steady-state destination in minimum time under $|\tau_m(t)| \leq \tau_{max}$ for all t. Express the optimal τ_m and ϕ as the step responses of stable systems (derive the transfer functions of these systems). What voltage u is required to generate this optimal torque? Express it as the step response of a stable system as well. Plot the optimal τ_m, u (call it u_{opt}), and ϕ (call it ϕ_{opt}) for $\tau_{max} = 0.1747$ [N m], $\phi_0 = 0$ and $\phi_f = \pi$ [rad]. In what time t_f this angle is reached in this case?

6. Now return to the system with ζ as in Table 1 and consider again the unity-feedback servo loop in Fig. 2(a) with C designed above. Selecting the reference signal as ϕ_{opt} (from the previous item), simulate the responses of u, ϕ (present on the same plot with ϕ_{opt}), and θ. Explain the behavior of ϕ.

7. Now consider the 2DOF servo loop in Fig. 2(b) with C designed above. What ϕ_{ref} and u_{req} shall be chosen to produce $\phi = \phi_{opt}$ (from the Item 5). Simulate the responses of u, ϕ (present on the same plot with the dashed ϕ_{ref}), and θ. Compare these plots with their counterparts in the previous item and explain the differences.

8. Now consider altering the design of ϕ_{ref} and u_{req} to avoid exciting the pendulum during moving the arm. The idea is to use posicast-like filtering of these signals to exclude exciting the pendulum mode. Consider a filter of the form

$$F_{posic}(s) = \alpha_1 + \alpha_2 e^{-\tau s}.$$

Find the parameters $\alpha_1 \in \mathbb{R}$, $\alpha_2 \in \mathbb{R}$, and $\tau > 0$ such that

   (a) $P_{\theta\phi} F_{posic}$ is an FIR (finite impulse response) system,
   (b) the static gain $F_{posic}(0) = 1$,
   (c) τ is the smallest possible delay guaranteeing all that

in symbolic form in terms of the parameters of the system. Plot the step response of the resulting $P_{\theta\phi} F_{posic}$. Also plot, at one graph, the signals ϕ_{opt} (dashed line) and $F_{posic}\phi_{opt}$ and, at another graph, the signals u_{opt} (dashed line) and $F_{posic} u_{opt}$, where ϕ_{opt} and u_{opt} are as derived in Item 5.

9. Replace now ϕ_{ref} and u_{req} used in Item 7 with their versions filtered by F_{posic} derived in the previous item. Simulate the responses of u, ϕ (present on the same plot with the dashed ϕ_{ref}), and θ (present at the same plot with the dashed θ from Item 7). Explain the behavior of θ and its difference from that in Item 7.

Remarks

- All plots should be presented on separate figures, unless explicitly stated otherwise.

- Use the time vector $t = \text{linspace}(0, 4.2, 1001)$ for all time-domain simulations, except those in Items 5 and 8. For simulations in Items 5 and 8 use $t = \text{linspace}(0, 1.8, 501)$.

- Present all time-domain angle plots in *degrees*, even though the models assume them in radians.

- It is advised to save plots using exportgraphics(gca,'name.pdf','ContentType','vector') from within your Matlab code (requires R2020a or newer). This saves plots as vector PDF files without white margins.

<div dir="rtl">

כללי ההגשה

יש לשלוח את הפרוייקט לכתובת: ravina.l@campus.technion.ac.il. קובץ ההגשה, ארכיון מסוג ZIP או RAR ששמו הוא שם המגישים, יכיל קובץ PDF (יש להגיש דוח מודפס; סריקה של כתב יד לא תתקבל) המתאר את התשובות לשאלות השונות, שלבי הפתרון וההסברים על פי הצורך. בנוסף, יכיל הארכיון את קבצי הקוד המקוריים, קבצי m.*, המבוארים הערות ומחולקים בהתאם לשאלות. יש לרכז את כל <u>החישובים בקובץ m.* יחיד</u>. את הפרוצדורות לציור הגרפים ניתן להגיש בקבצי m.* נוספים.

</div>